When the Old Meets the New: Examples of What Established Analytical Methods Look Like in a Modern Computer Environment

Mr Jon Aas
CT Innovations Ltd

Abstract

There is a vast number of analytical solutions of Ordinary and Partial Differential Equations (ODE and PDE) available in engineering literature, books, journals and teaching material, starting from the definition of the Partial Differential Equation (PDE) for thin plates formulated by Lagrange in 1811 /1/ see figure 1, the mathematical breakthroughs by Augustin-Louis Cauchy in early 1800s /2/ and Claude-Louis Navier /3/ using double Fourier series to solve the problem of a simply supported plate with different types of loads in 1820. The focus in this paper is the analytical solution of rectangular plates.

To solve these PDEs by hand calculations were the norm, limiting the practical use of these mathematical findings significantly. Today's engineers and designers working in product development have vast computer resources available to them to implement these PDEs for better understanding of the behaviour of rectangular plates. The introduction of Formulation, Validation and Verification in product development has actualised the analytical solutions, as numerical solutions computed using FEA technology must be compared against "exact solutions" for verification.

It is pertinent to ask the question: "What would the forefathers in Classic Solid Mechanics have done if they had our computer resources available to them?" A number of examples are made to show what effective use of state-of-the-art computing can do to revive the classical methods.

1. Conclusions

We owe it to the originators of the underlying mathematics for Classical Solid Mechanics to demonstrate what they could have done themselves if they only had the same computer technology as we have. There are no limits to the opportunities available today to advance the understanding of mathematical models, i.e., this is a Blue-Sky Opportunity.

What is explored here is what we currently can do in June 2019. However, computer technology is moving forward at an accelerating speed. Using computers the same way as in the 1960s and 1970s doesn't measure up. We must use computers the way the computer game generation is accustomed to. We must move faster than in the last two decades to keep up with the pace of change, let alone catch up on the advances made. Established thinking and working practices must be replaced by new paradigms for shape representation

/28/ and new software development in user interfaces, computer graphics and multi-computer job administration, in the use of relational databases, application scripting and most of all in exploiting the advances in High Performance Computing (HPC).

As an example, the first Exa Floating Point Operations Per Second (FLOPS) computers are predicted available in 2021 /29/ computers that can do 1,000,000,000,000,000,000 FLOPS. To exploit the enormous computer power soon available at affordable prices, the established practices of **"ONE big analysis"** have to be replaced by **"MANY smaller analyses"** in Formulation, Validation & Verification based working practices, combining analytical and numerical analyses. Scaling and tuning of the technology will in time create **"MANY big analyses"**. Many-core heterogeneous parallel computers will open up for parallel processing at an unimaginable scale, compressing the clock wall time so simulations that took weeks instead become interactive. **What do you plan to do today so you are ready when Exa FLOPS machines arrive?**

I for one have ambitions in this respect: I want Pica FLOPS, or better, Exa FLOPS, computer power hanging off the tip of my games controller, with the software functionality to compute, capture and present the results coming out of the multi-simulations in real-time. Most of the computer technology is here already, **when will software for mathematical modelling catch up?**

2. References

There is an unending list of possible references for the topics covered in this paper. Instead of selecting an arbitrary reference in a long list of possibilities, here is a modern way of directing readers to authoritative sources for further reading: to use the Internet, more specifically Wikipedia as the knowledge resource. Wikipedia gives a clear and concise presentation of most topics with a list of references for further reading.

/1/ about Lagrange: https://en.wikipedia.org/wiki/Joseph-Louis_Lagrange

/2/ about Augustin-Louis Cauchy: https://en.wikipedia.org/wiki/Augustin-Louis_Cauchy

/3/ about Claude-Louis Navier: https://en.wikipedia.org/wiki/Claude-Louis_Navier

/4/ Fung, Y. C., Tong, Pin, Chen Xiaohong (2017) "Classical and Computational Solid Mechanics", Second Edition, World Scientific Publishing Co. Pty. Ltd.

/5/ Reddy, J. N. (2007), "Theory and Analysis of Elastic Plates and Shells", Second Edition, CRC Press, Boca Raton, FL, USA,Chapter 3, p95-123.

/6/ Timoshenko, Stephen P. and Woinowsky-Krieger, S. “Theory of Plate and Shells”, Second Edition, McGraw-Hill, article 28, pp 108-110.

/7/ Bittner, Ernst: (first edition 1938, second edition 1965) “Platten und Behälter”, Springer Verlag, Wien.

/8/ Rűdiger-Urban (1955), “KreisZylinderscalen - Circular Cylindrical Shells”, B. G. Teubner Verlagsgesellschaft Leipsig.

/9/ About Kelvin Functions: https://en.wikipedia.org/wiki/Kelvin_functions

/10/ Holand, Ivar, (1961), “Tables for the Analysis of Cylindrical Tanks or Tubes with Linearly Variable Thickness”, International Association for Bridge and Structural Engineering, Zurich.

/11/ Gibson, J. E. (1954), “The Design of Cylindrical Shell Roofs”, E & F N Spon Ltd., London

/12/ Gibson, J. E. (1960), “The Design of Cylindrical Shell Roofs”, E & F N Spon Ltd., London

/13/ Gibson, J. E. (1975), “Computing in Structural Engineering”, Applied Science Publishers Ltd, London

/14/ About Fortran: https://en.wikipedia.org/wiki/Fortran

/15/ Timoshenko, Stephen P. and Woinowsky-Krieger, S. “Theory of Plate and Shells”, Second Edition, McGraw-Hill.

/16/ Flűgge, Wilhelm (1960), “Stresses in Shells”, Third Printing, Springer-Verlag New York Inc.

/17/ Scabó, Barna and Babuška, Ivo, (2011), “Introduction to Finite Element Analysis, Formulation, Verification and Validation”, John Wiley & Sons, Ltd, UK

/18/ ASME V&V 10-2006, Reaffirmed 2016, “Guide for Verification and Validation in Computational Solid Mechanics”, The American Society of Mechanical Engineers, New York.

/19/ ASME V&V 10.1-2012, “An Illustration of the Concepts of Verification and Validation in Computational Solid Mechanics”, The American Society of Mechanical Engineers, New York.

/20/ Blezinger, Kai-Uwe, (2008) “Theory of Plates, Part 2: Plates in Bending”, Lecture Notes, Winter Semester 2012/2013, Lehrstuhl Fur Statik, Technische Universitat Munchen, Deutschland, p8.

/21/ Roarche, Patrick J. (2009), “Fundamentals of Verification and Validation”, Hermosa Publishers, Albuquerque, NM.

/22/ Timoshenko, Stephen P. and Woinowsky-Krieger, S. “Theory of Plate and Shells”, Second Edition, McGraw-Hill, Article 22, p83-87.

/23/ Timoshenko, Stephen P. and Woinowsky-Krieger, S. “Theory of Plate and Shells”, Second Edition, McGraw-Hill, Chapter 7, Articles 52-56, p229-258

/24/ Lusas: http://www.lusas.com/

/25/ About Pivot Tables: https://en.wikipedia.org/wiki/Pivot_table

/26/ About Analytics: https://en.wikipedia.org/wiki/Analytics

/27/ About ParaView: https://en.wikipedia.org/wiki/ParaView and ParaView official site: https://www.paraview.org/

/28/ Aas, Jon (2019), “Dislocation Meshing, a Credible Solution to Automatic Hexahedral Meshing”, NAFEMS World Congress & SPDM Conference 2019, Quebec, Canada.

/29/ about Exa FLOPS computing: https://en.wikipedia.org/wiki/Exascale_computing