When the Old Meets the New:

Examples of What Established Analytical Methods Look Like in a Modern Computer Environment

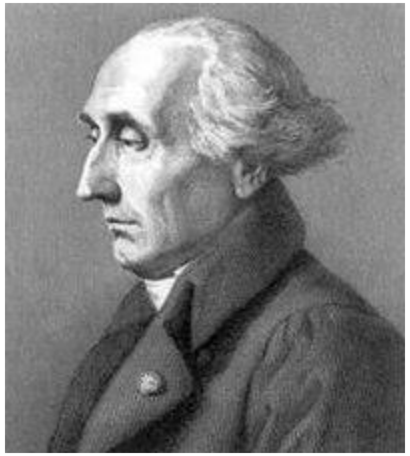
Jon Aas, CT Innovations Ltd

The presentation

- The Idea
- Examples
- An Engineering Workbench Prototype
- Requirement Specification for …
- Examples
- A Blue Sky Opportunity
- Conclusion

When the Old meets the New …

The Idea



Joseph-Louis Lagrange (1736-1813)

Joseph Fourier (1768-1830)

Augustin-Louis Cauchy (1789-1857)

Claud-Louis Navier (1789-1836)

Maurice Levy (1838-1910)

The Old:

- Substantial amount of theoretical mathematics developed over the last 200 years
- Including analytical methods for solving Partial Differential Equations (PDE) for engineering problems like rectangular plates
- They had few, if any, practical tools for solving the PDEs and using their solutions for advancing engineering problems

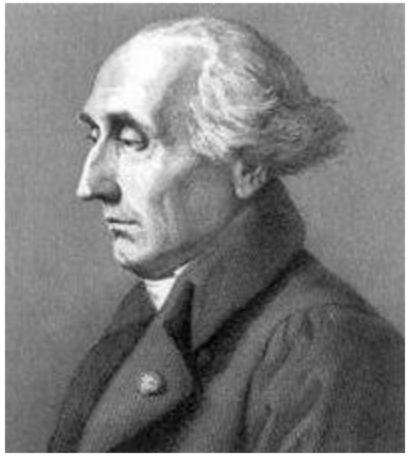
The New:

- Modern computing has evolved substantially over our life-time
- the rate of change in the development of computers and software tools is accelerating
- New paradigms in user interfacing, data handling and computer power are transforming the way we will work in the near future and beyond.

The **first** group of questions includes: “What if … ?”
What would the old mathematicians and engineers who developed the PDE theory and solution methods have done today if they had the computing technology that we have?

When the Old meets the New …

The Idea



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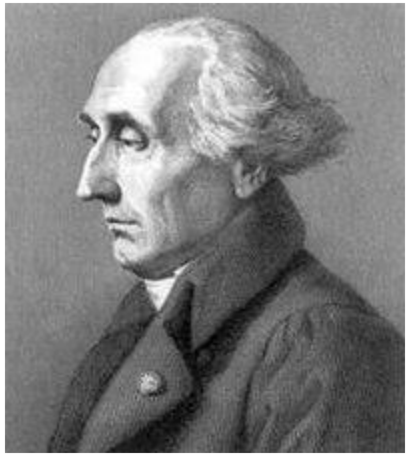

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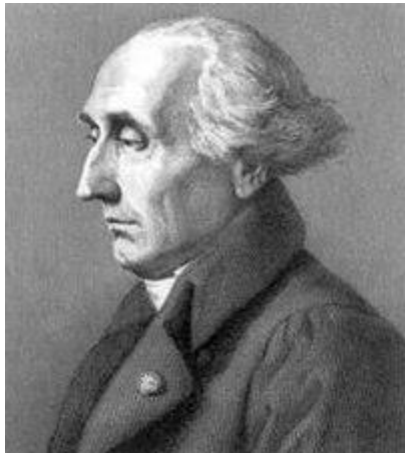

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The Idea: Verification & Validation

Relative Error:

$$\frac{\left|\phi_i(u_{EX})-\phi_i(u_{FE})\right|}{\left|\phi_i(u_{EX})\right|}\leq \tau_i$$

Exact solutions: in short supply

Approximate solutions: in plentiful supply

Where

- $\phi_i u_{EX}$ is the exact solution for a particular Result of Interest
- $\phi_i u_{FE}$ is the approximate solution from a numerical analysis for a particular Result of Interest
- i is an index for the number of Results of Interest for consideration, for example maximum displacement, temperature, stress, etc. at a particular point, curve, surface, volume
- τ_i is the relative error for the FE analysis compared with the EXact solution

The **second** group of questions includes:

If we make use of the mathematical theory for Classical Solid Mechanics to develop modern computer tools for engineers, what should they do and what will they be used for?

Or more pertinently, we know there is a large gap in the provision of “Exact Solutions” for use of Verification & Validation working practices. How can this gap be filled?

The Idea: Verification & Validation

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The Idea: How can we prepare for the future?

There is a need for a practical tool for calculating analytical and numerical (FEA) results:

- To enable practicing engineers to understand the behaviour of plates so they can design better structures;
- To encourage the practical side in teaching of mathematical modelling at universities;
- To motivate youngsters (read: gamers) looking for a meaningful and interesting career to become mathematical modellers, ref. (STEM – Science, Technology, Engineering Mathematics) https://en.wikipedia.org/wiki/Science,_technology,_engineering,_and_mathematics

The **third** group of questions includes :

When we can find a “modern” way of making full use of the theoretical work from the 19th and 20th centuries, how can we prepare the future of engineering simulation?

These are the issues that have motivated the research effort reported here.

When the Old meets the New …

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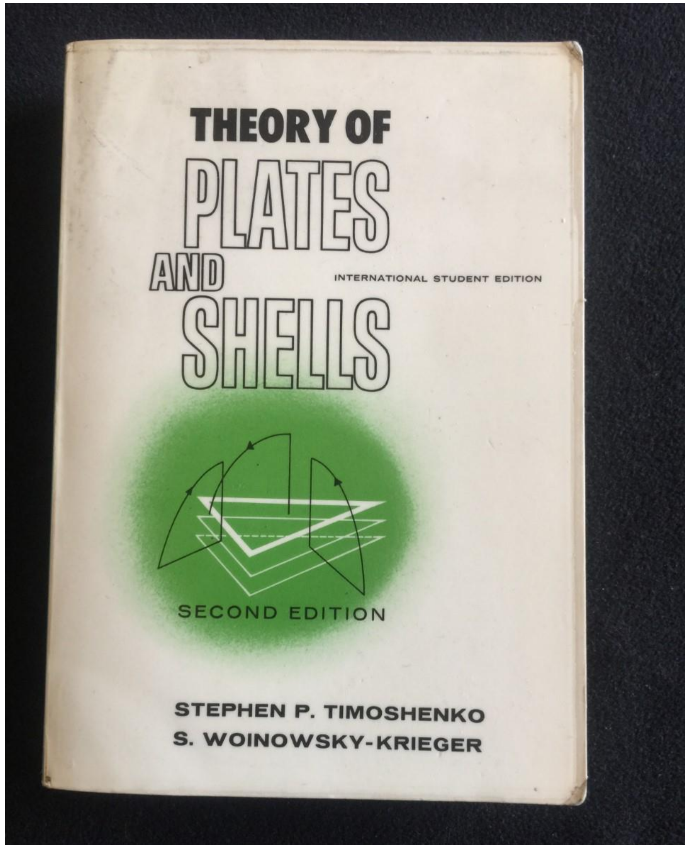
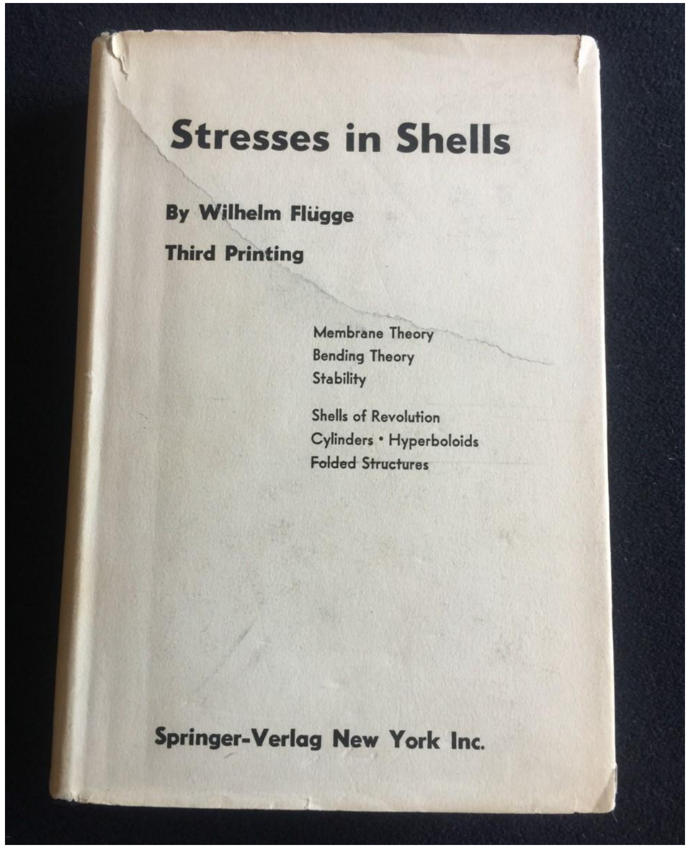
Examples: Sources of the theoretical work





Theory of Plates and Shells By Stephen P. Timoshenko & S. Woinowsky-Krieger

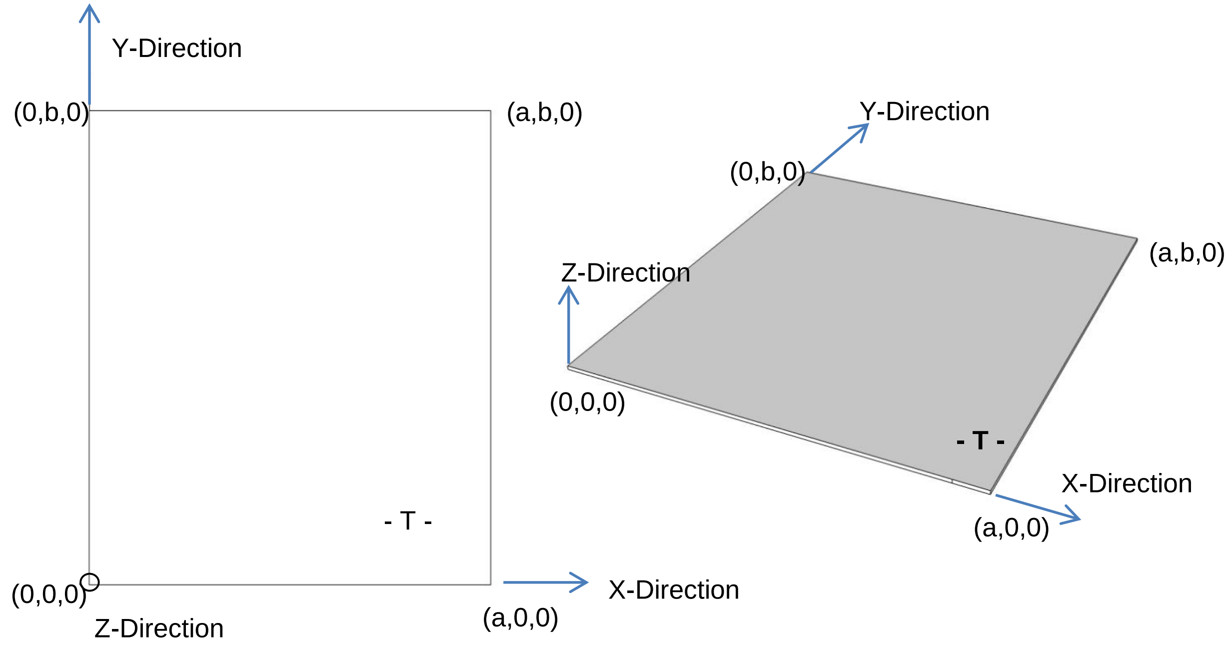
How many printing mistakes have you found to date?

"Stresses in Shells" By Wilhelm Flügge

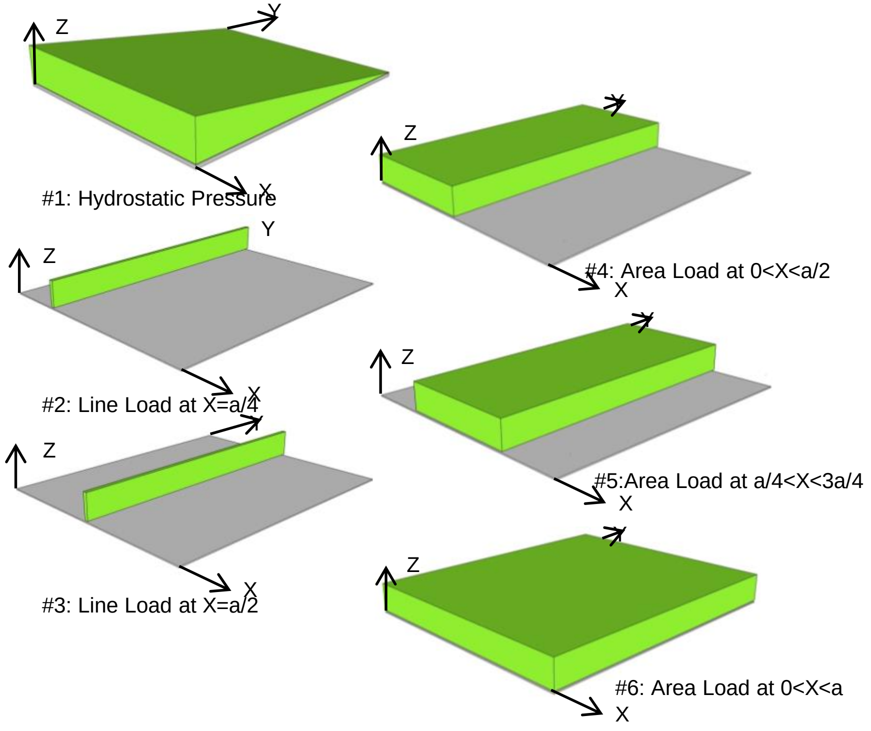
What do your copies look like?

When the Old meets the New …

The Idea: using Rectangular Plates to explore …



A typical Rectangular Plate



A selection of load types for
- Navier's solution method
- Levy's solution method when the Fourier series is used in x-direction.

In this presentation focus will be on rectangular plates based on material found in "Theory of Plates and Shells" by Stephen P. Timoshenko & S. Woinowsky-Krieger and others …

When the Old meets the New …

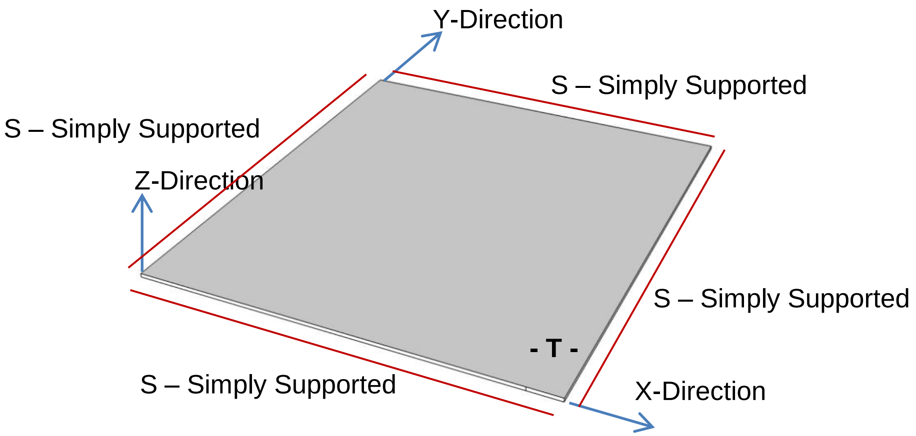
Examples: Classic Plate Theory (CPT)

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w\,\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \quad \text{or} \quad \nabla^4 w = \frac{q}{D} \quad \text{where}$$

$$q = \mathrm{f}(x, y)$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

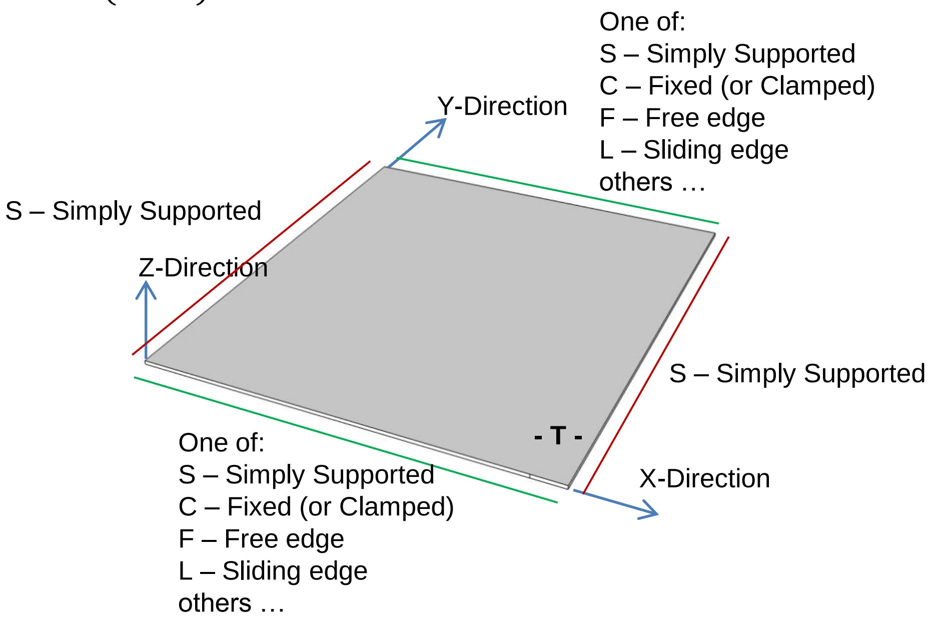
This is the bi-harmonic PDE governing rectangular plates with even thickness and small deformations, no shear effect included, (Kirchhoff-Love formulation), called the Classic Plate Theory (CPT).



$$w(x,y) = \frac{1}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_{mn}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\, sin\,\frac{m\pi x}{a}\, sin\,\frac{n\pi y}{b}$$

a_{mn} is specific for each load type

Claude-Louis Navier’s solution method from 1820



$$w(x,y) = \sum_{m=1}^{\infty} W_m(y)\,\sin\frac{m\pi x}{a}$$

$W_m(y)$ is a fourth order ODE

The Fourier series is used between X=0 and X=a

Maurice Levy’s solution method from 1899

This is the one you will find in “all the books”

Examples: Maurice Levy's solution method …

On Maurice Levy's solution from 1899:

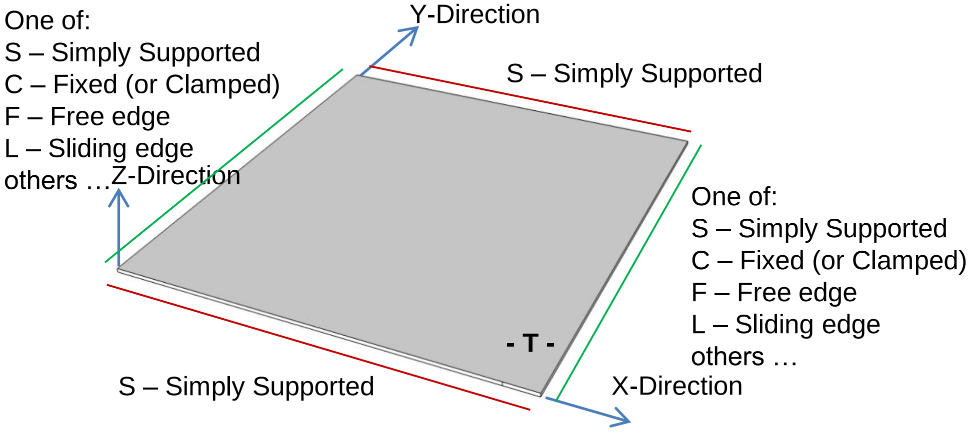
Maurice Levy's solution method is a major step forward as it transforms the Partial Differential Equation (PDE) into an Ordinary Differential Equation (ODE) in one variable:
This can be done in either x or in y. The transformation into an Ordinary Differential Equation (ODE) in $W_n(y)$ is shown on the righthand side in the previous slide and the similar transformation in $W_n(x)$ is shown on this slide.

There are a number of methods available to solve a forth order Ordinary Differential Equation (ODE). T & WK suggest a simplified approach where the coordinate system is shifted to mid-plate in y-direction with a choice of evenly distributed load and simply supported boundary condition on all four edges. It works, and they enumerate different load types working through the theory of different solution functions. How can this approach be generalised ?

My own lecturer at uni directed us in our 3rd year project to use the same method as for the PDE for circular shells, avoiding the introduction of restrictions to position of the coordinate system, the load type and boundary conditions.
Dr J. N. Reddy suggests two alternative solutions, one being the same as in T&WK.

Additionally, solving the ODE ends up with specific functions for $W_n(x)$ or in $W_n(y)$ (the Cs are found by applying the alternative boundary conditions to the fourth order ODE) dependent on the combination of boundary conditions on the two other opposite edges. The result is enumeration of cases using codes like SSSS, CCSS, etc for the plate boundary conditions, combining only the first three cases from table two slides ahead: S (simply supported), C (clamped) and Free (F).
The superposition principle is used to alter the boundary condition on the two simply supported edges by adding a bending moment and/or a shear force at these edges. In this way a simply supported edge can become a fixed edge and anything in between.

However, there are large gaps in the literature on the combination of load types and boundary conditions for solving the CPT.



$$w(x,y) = \sum_{n=1}^{\infty} W_n(x) \sin\frac{n\pi y}{b}$$

$W_n(x)$ is a fourth order ODE

The Fourier series is used between Y=0 and Y=b

Maurice Levy's solution from 1899

More on Maurice Levy's Solution Method will be covered in the next presentation to celebrate the 120th anniversary for the publication of his solution method …

Examples: The typical coverage in literature

The bi-harmonic PDE known as "CPT"

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w\,\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

An open-ended number of solutions, q(x,y)

The Navier's bi-directional Fourier series solution method, all edges simply supported

$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} sin\,\frac{m\pi x}{a}\, sin\,\frac{n\pi y}{b}$$

There are other boundary conditions that other solution methods are capable to describe

a_{mn} for the uniformly distributed full load

$$a_{mn} = \frac{4qo}{ab} \int_0^a \int_0^b \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}\, dxdy = \frac{16qo}{\pi^2 mn}$$

There are other load types that are also described with an a_{mn}

The maximum deflection at the centre of a rectangular plate

$$w_{max} = \frac{16qo}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^{\frac{m+n}{2} - 1}}{mn\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

What about the deflection across the plate surface? What about all the other result components that are needed for full assessment of performance?

"Satisfactory approximation is obtained by taking only the first term in the series for a square plate"

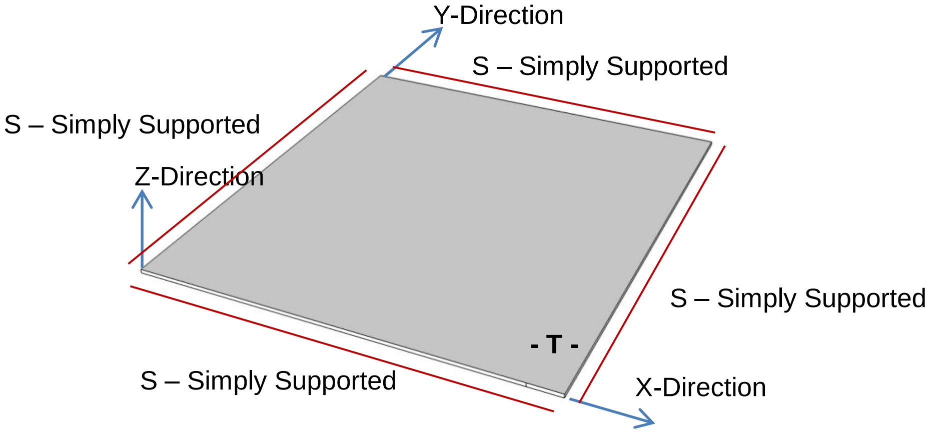
$$w_{max} = \frac{4qoa^4}{\pi^6 D} = 0.00416\frac{q_o a^4}{D}$$

One term in the series? Any numerical evidence to support this? The derived quantities converge slower and at different rates, one term is not enough …

This example is taken from pages 109-110 in T&WK and is typical: "… a reader can easily fill in the gaps."

Examples: The result components for the CPT simply supported plate

The Navier's solution method:



$$w(x,y) = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

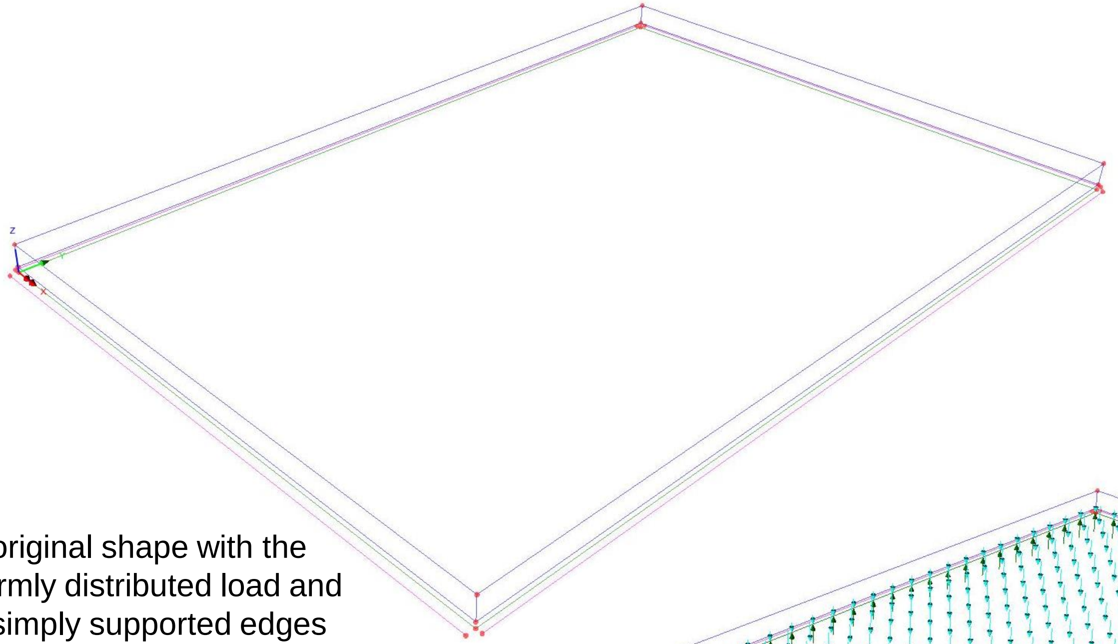
a_{mn} is specific for each load type

w - deflection	w
w,x - rotations	$\frac{\partial w}{\partial x}$
w,y - rotations	$\frac{\partial w}{\partial y}$
Mxx – bending moment	$M_{xx} = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$
Mxy – bending moment	$M_{xy} = D\,(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$
Myy – bending moment	$M_{yy} = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$
Qx – transverse shear force	$Q_x = -D\left[\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right]$
Qy – transverse shear force	$Q_y = -D\left[\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}\right]$
Vx – effective shear force	$Vx = D\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2}\right]$
Vy – effective shear force	$Vy = D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right]$

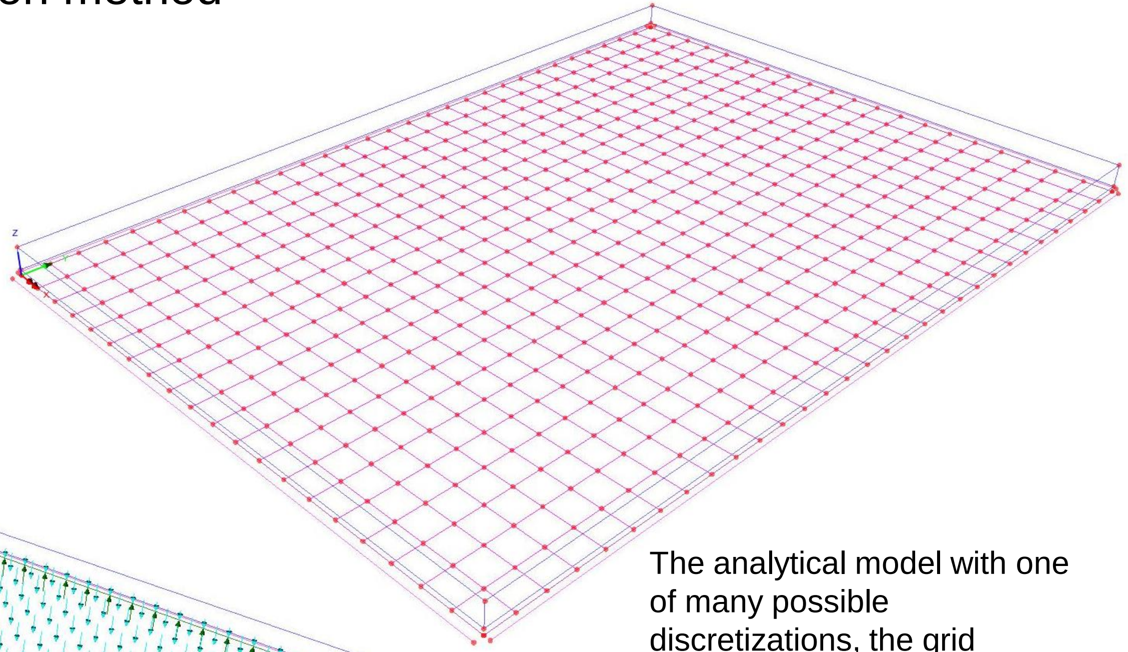
There are ten result components that have a physical interpretation for a rectangular plate. Their general mathematical representation is given on the righthand side. Their specific mathematical representation depends on the solution method used, i.e., the functions used for w, i.e., the solution method chosen.
For an extended range of boundary conditions, all ten result components are used to define the terms the PDE has to satisfy on the boundaries.
The choice of w(x,y) determines what boundary conditions can be used when solving the PDE.

The w(x,y) for Navier's solution method is given on the left, restricts all edges to be simply supported.
For the Navier's solution method the a_{mn} is specific for each load type.

Examples: Result components using Navier's solution method



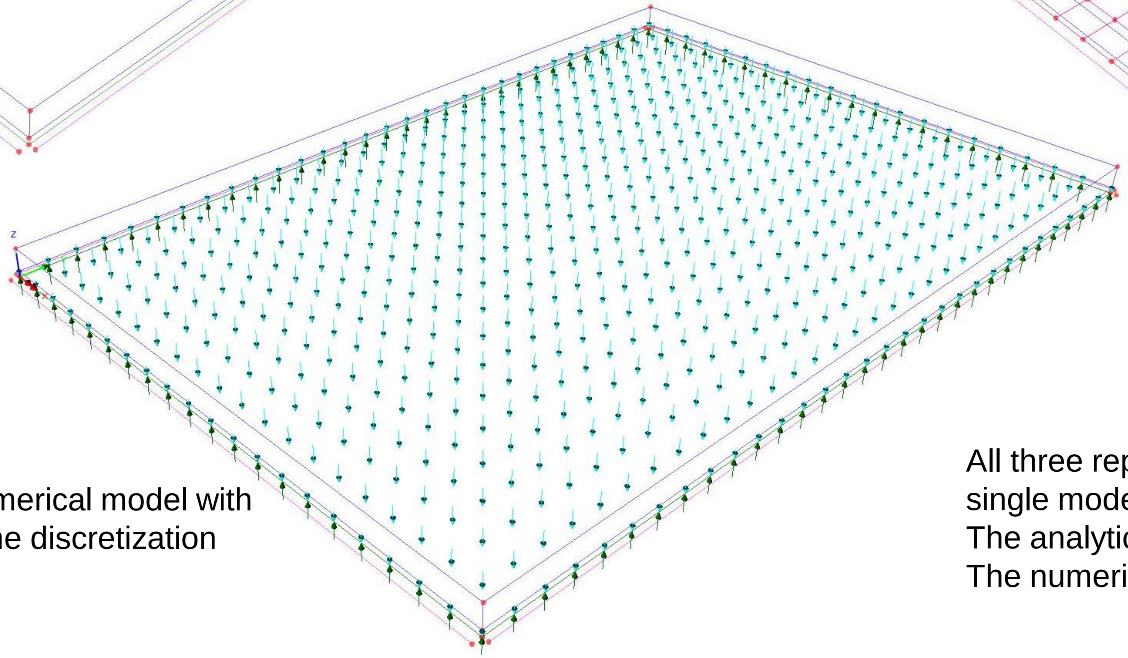
The original shape with the uniformly distributed load and four simply supported edges



The analytical model with one of many possible discretizations, the grid

Full uniformly distributed load

$$a_{mn} = \frac{16qo}{\pi^2 mn} \qquad (\text{m,n} = 1,3,5,\ldots)$$

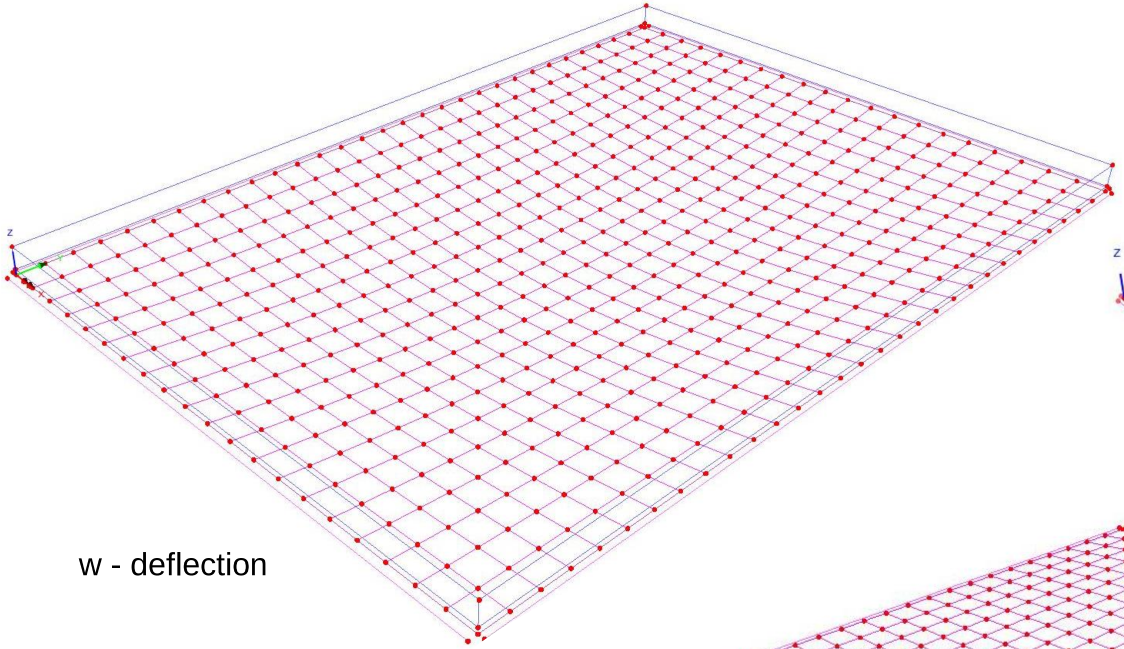


The numerical model with the same discretization

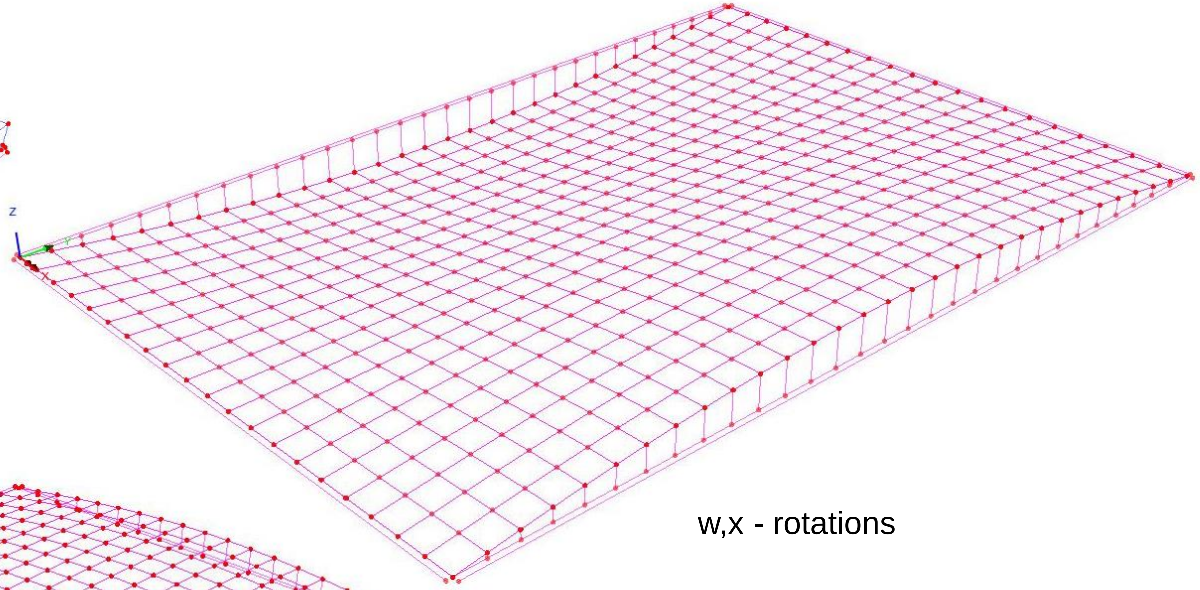
All three representations may be held together as a single model in the FEA pre-processor.
The analytical results can be added as a deformed grid
The numerical model can be computed as an FEA model

When the Old meets the New …

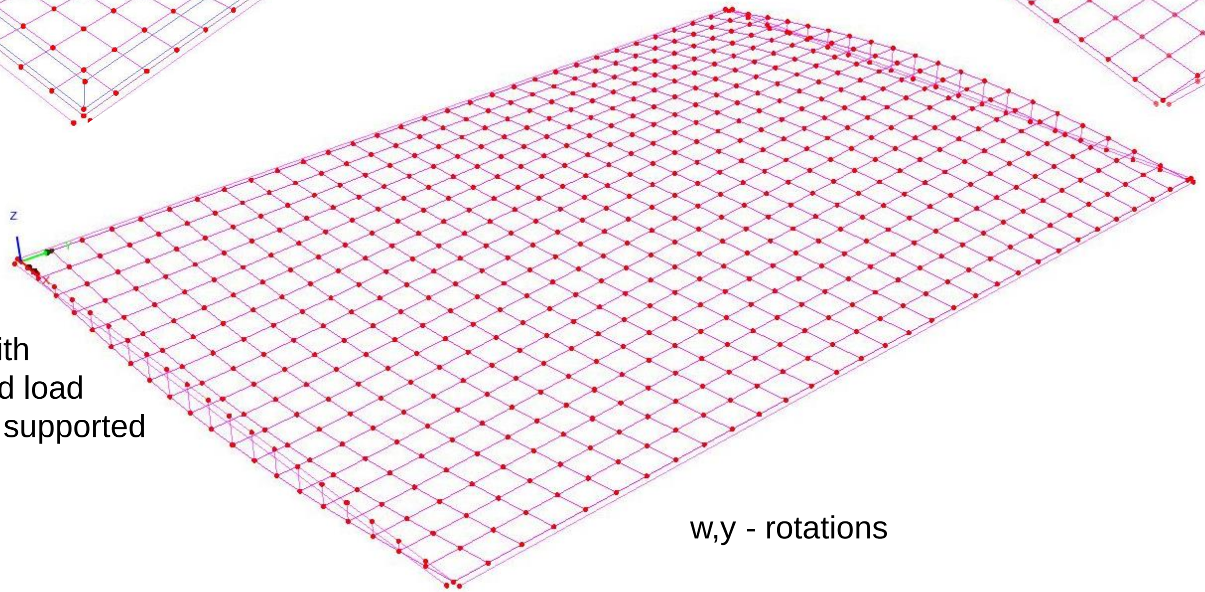
Examples: Result components using Navier's solution method



w - deflection



w,x - rotations

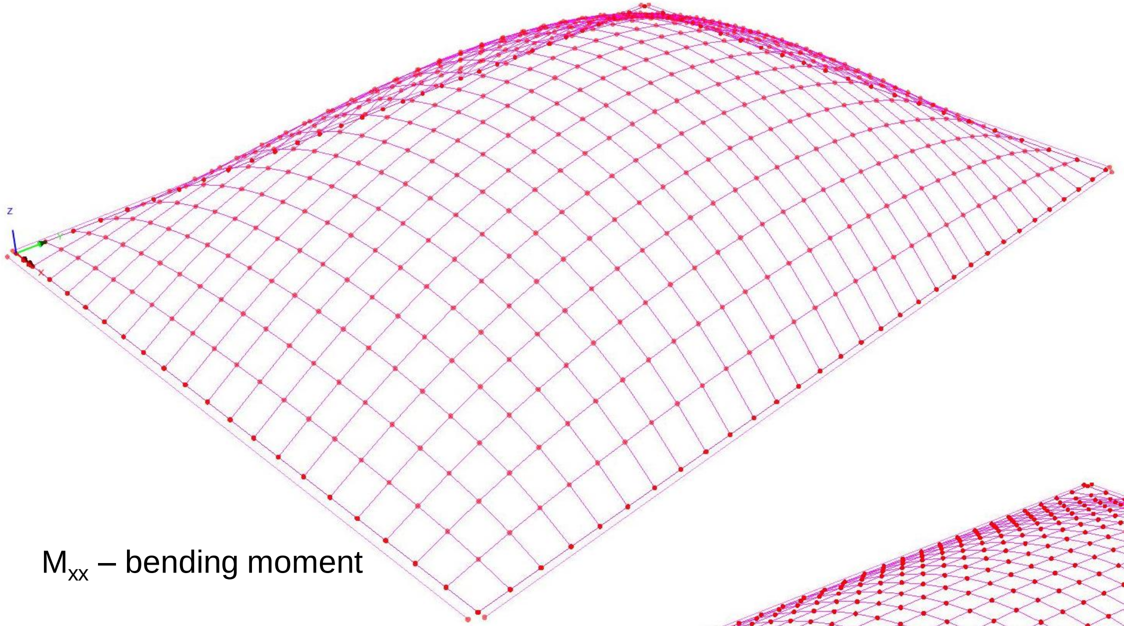


w,y - rotations

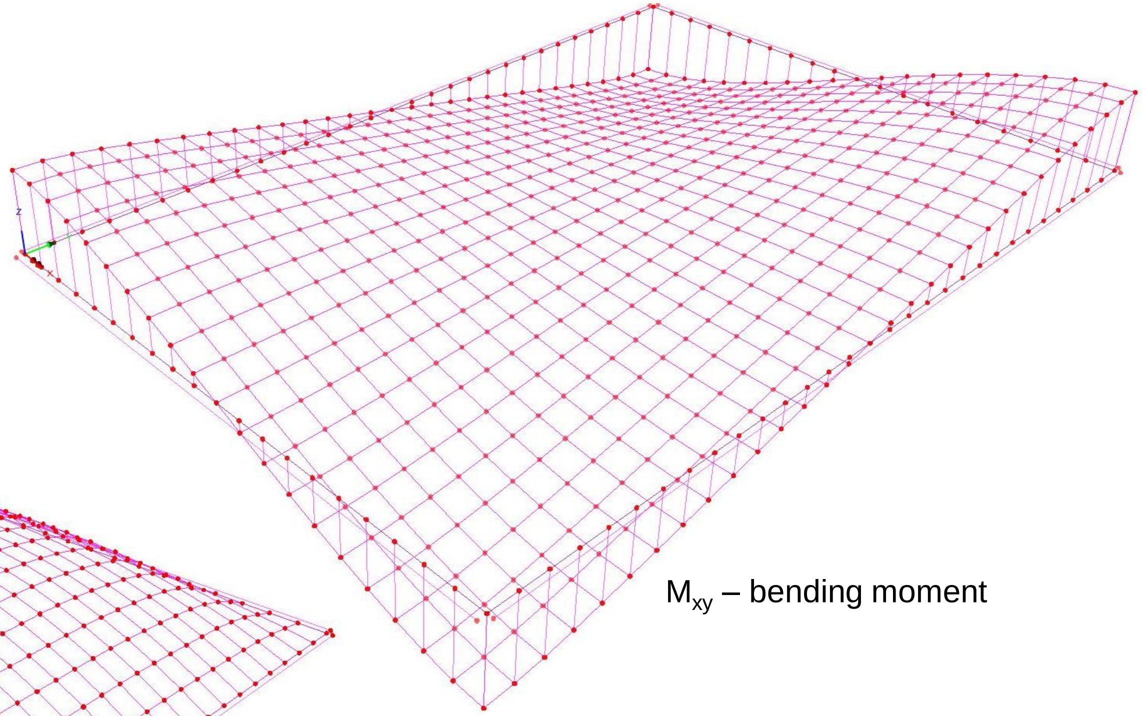
One of many discretizations with
- Load: full uniformly distributed load
- Boundary Conditions: simply supported

When the Old meets the New …

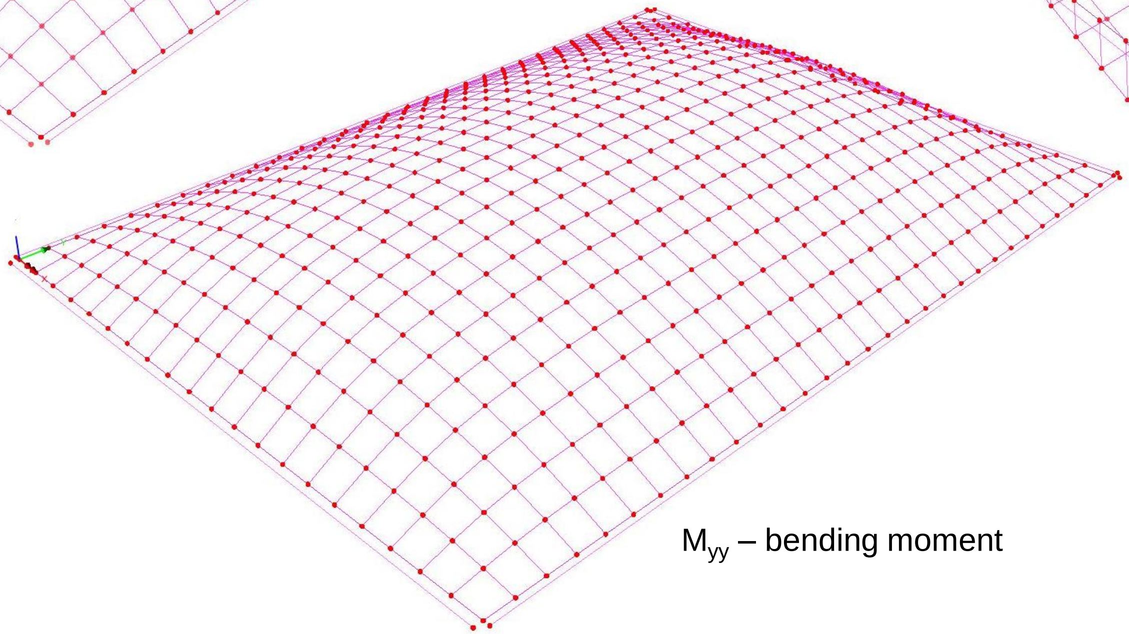
Examples: Result components using Navier’s solution method



M_{xx} – bending moment

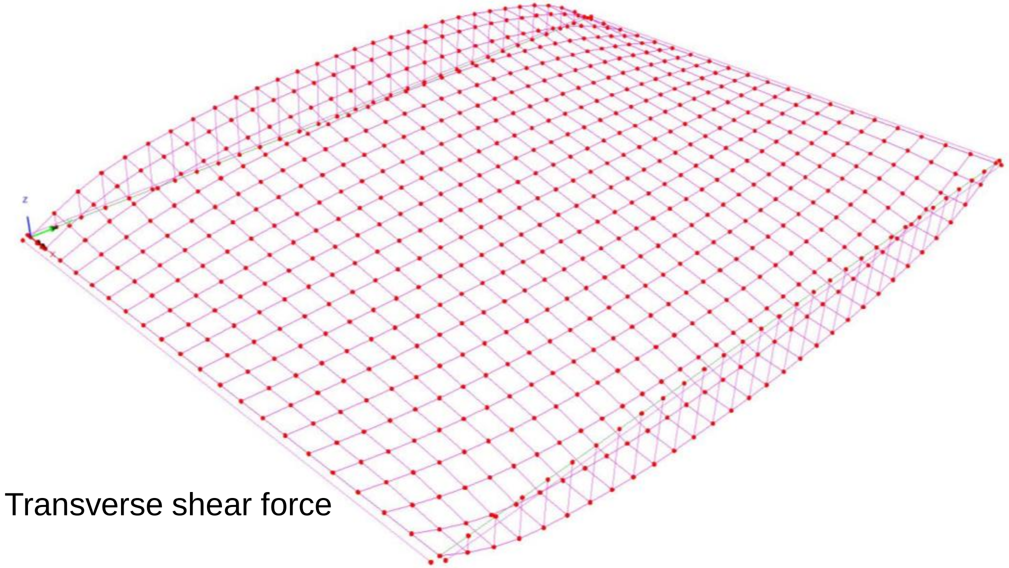


M_{xy} – bending moment

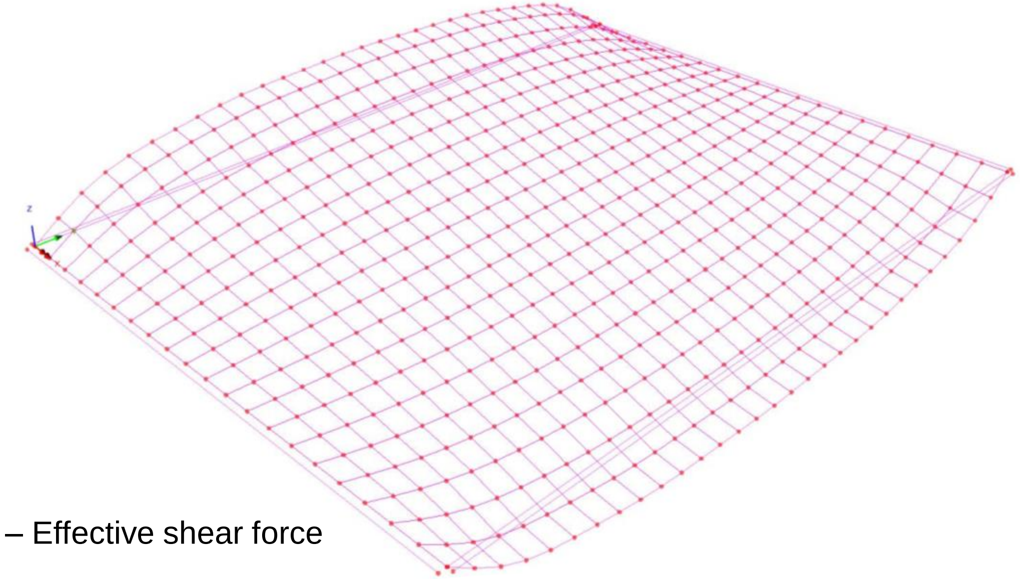


M_{yy} – bending moment

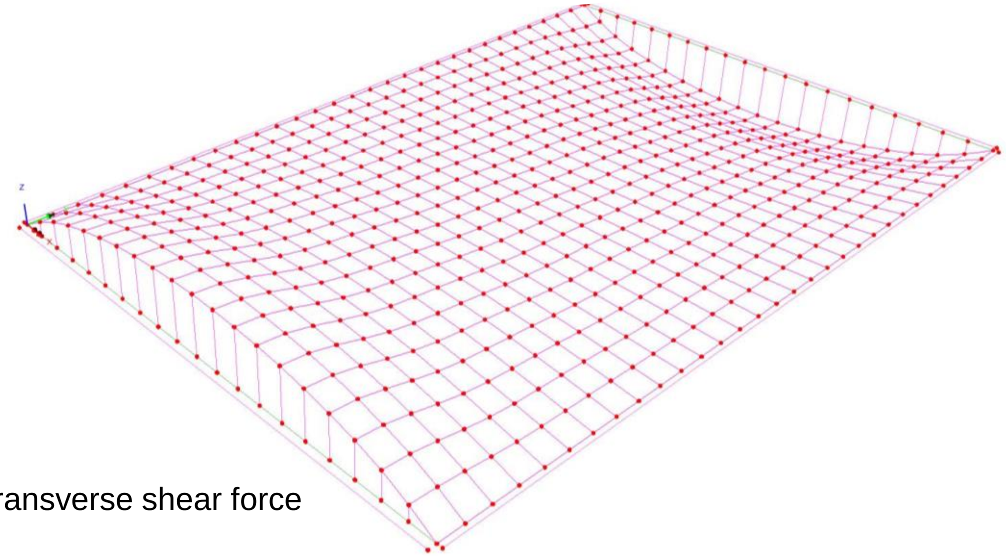
Examples: Result components using Navier’s solution method



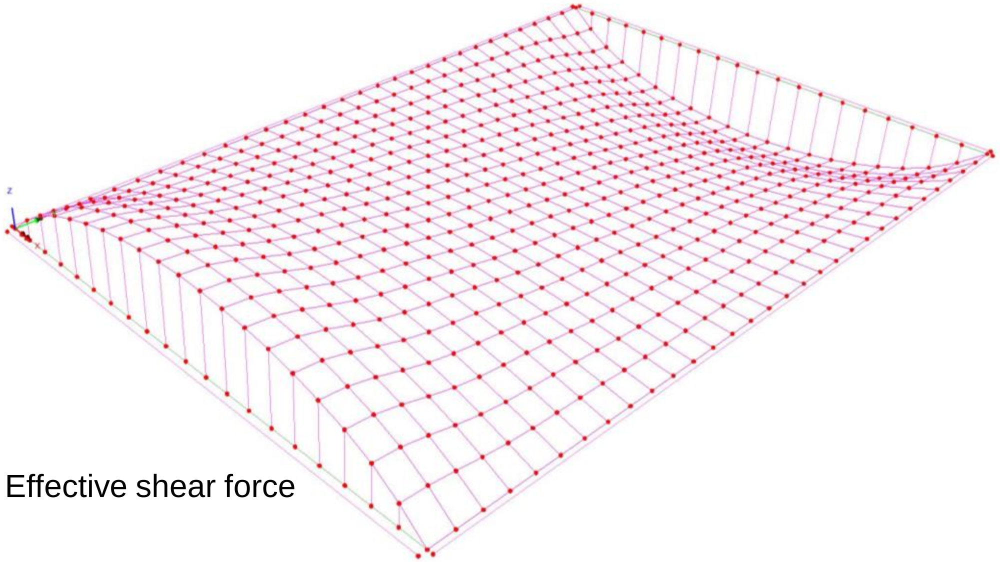
Q_x – Transverse shear force



V_x – Effective shear force



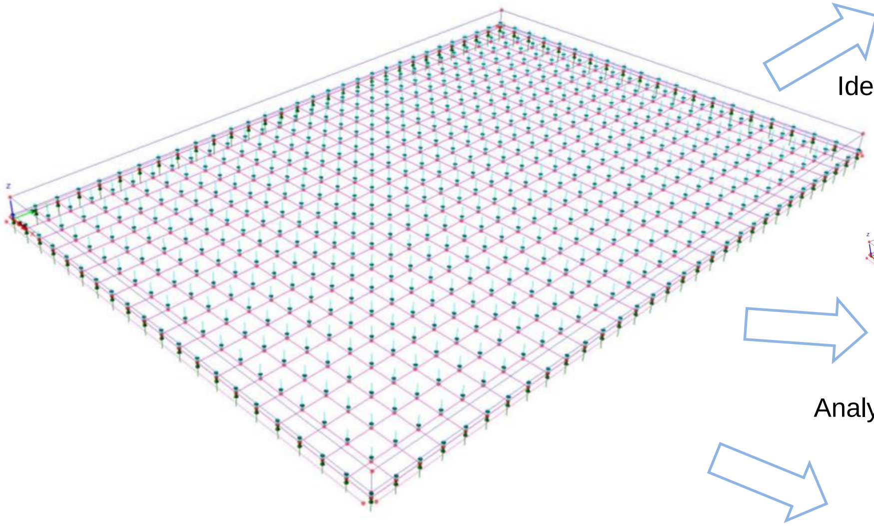
Q_y – Transverse shear force

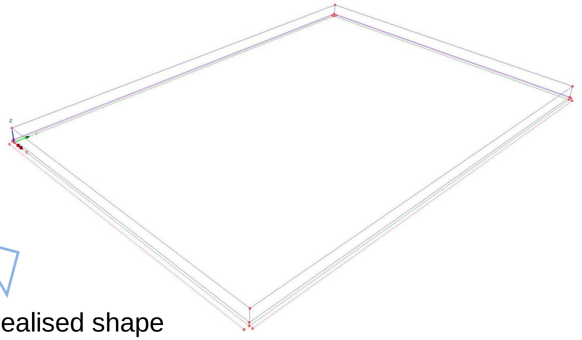


V_y – Effective shear force

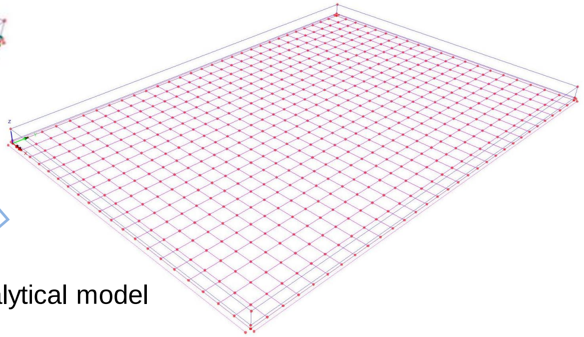
Examples: Result components using Navier's solution method

The combined model representation



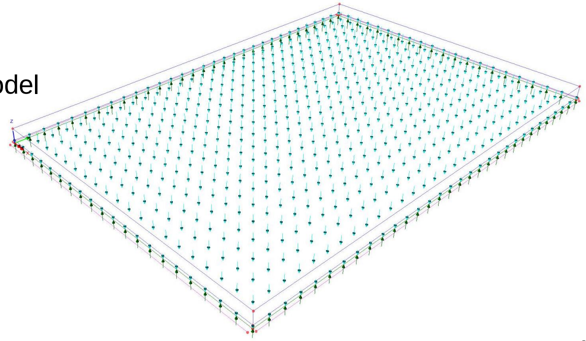


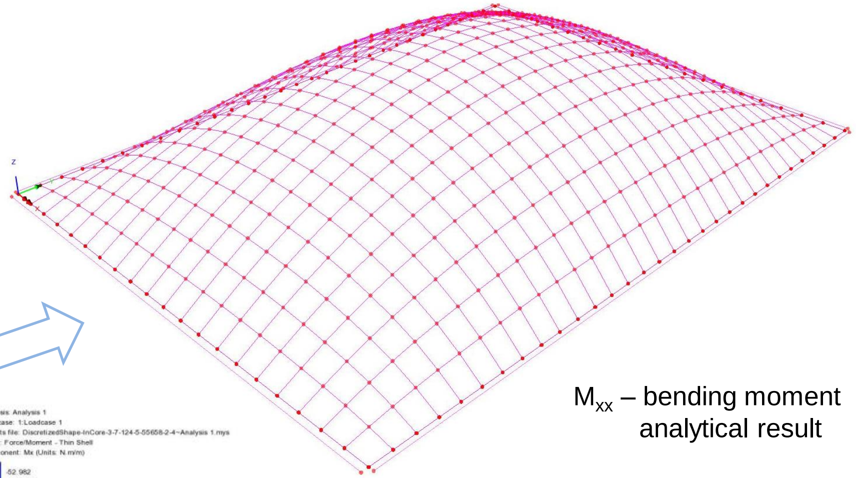
Idealised shape



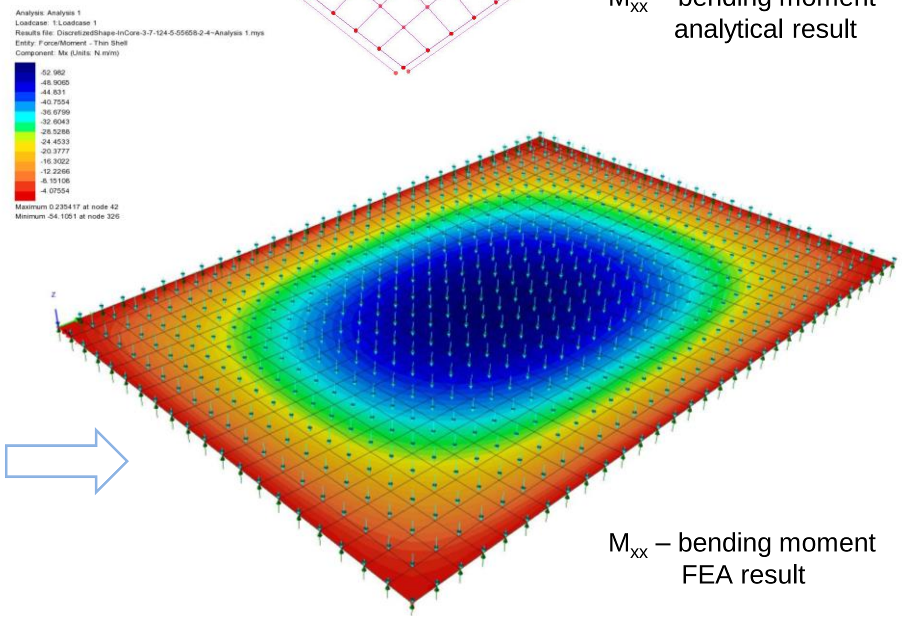
Analytical model

FEA model





M_{xx} – bending moment analytical result



M_{xx} – bending moment FEA result

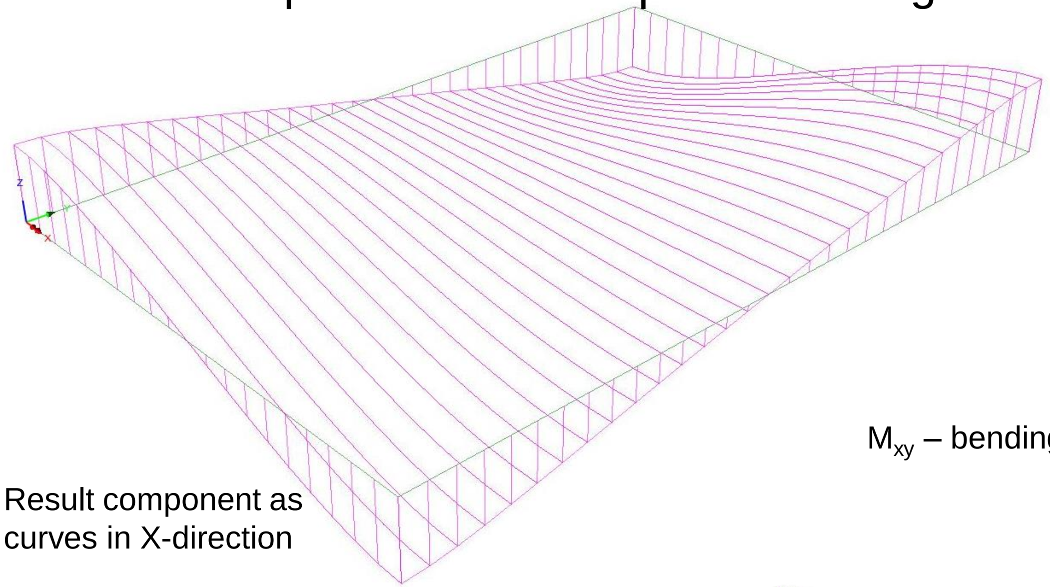
The idealised shape, the analytical model and the FEA model co-exist in the FEA pre-processor

This applies to all 10 result components

The results can be computed directly for the FEA model

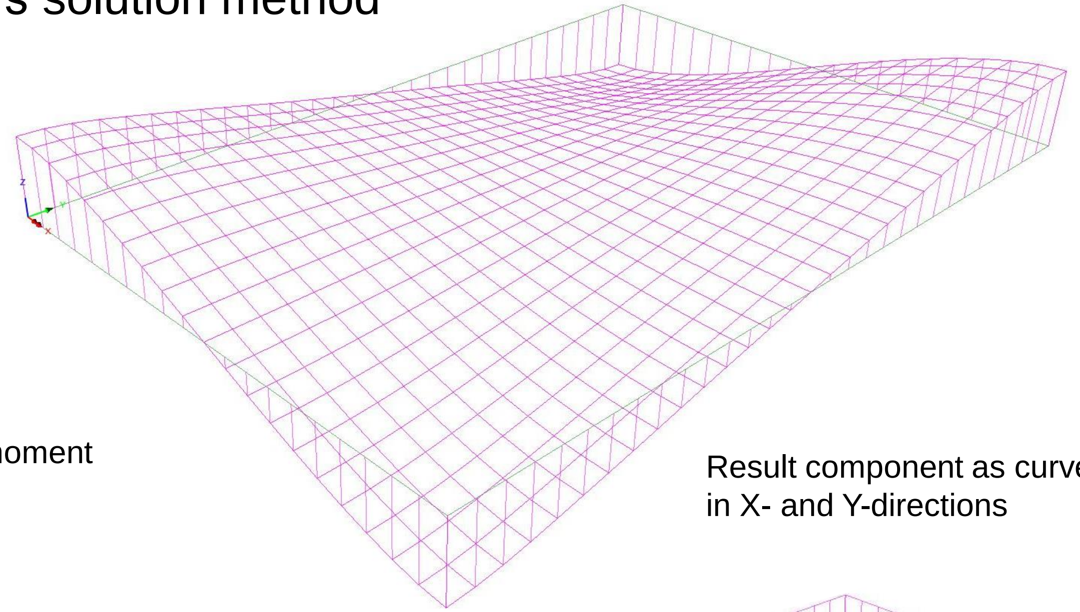
When the Old meets the New …

Examples: Result components using Navier’s solution method

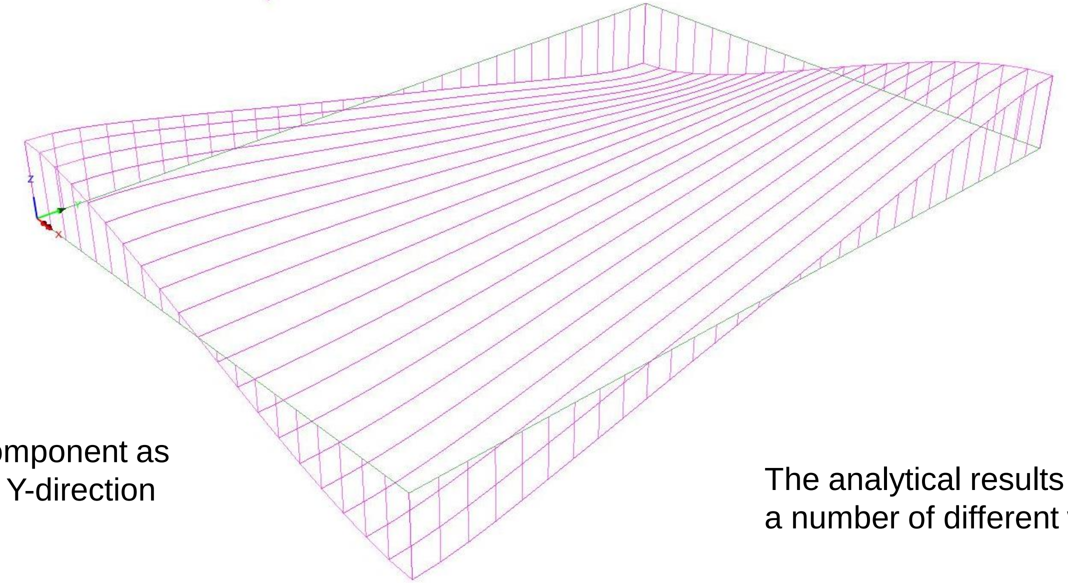


Result component as curves in X-direction

M_{xy} – bending moment

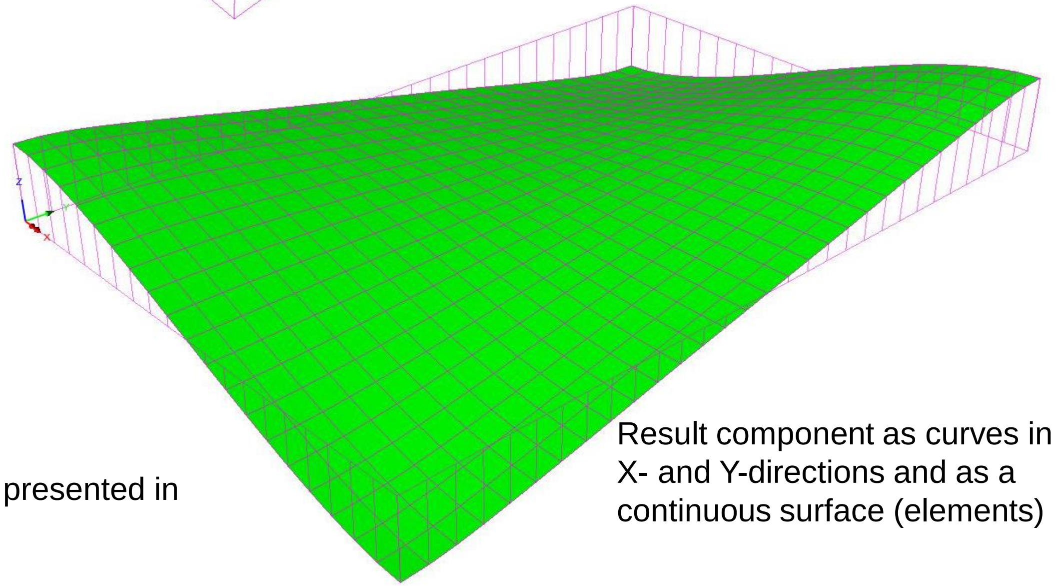


Result component as curves in X- and Y-directions



Result component as curves in Y-direction

The analytical results can be presented in a number of different ways

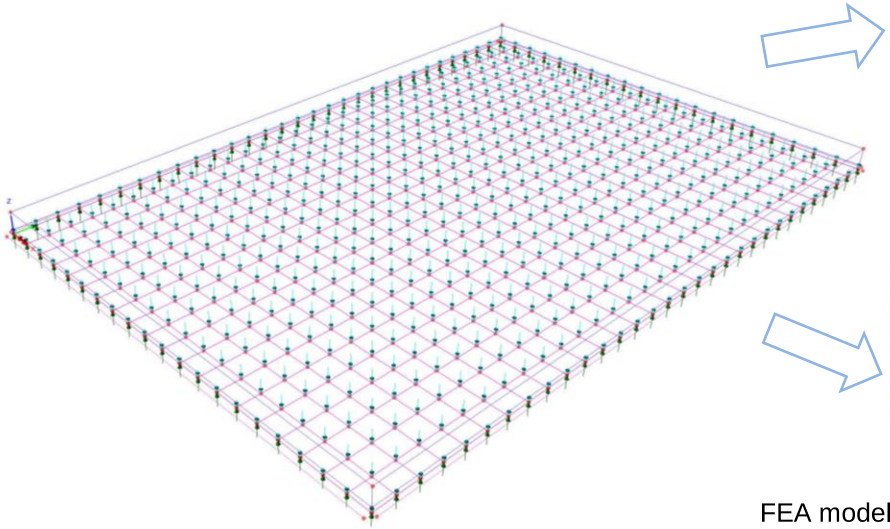


Result component as curves in X- and Y-directions and as a continuous surface (elements)

When the Old meets the New …

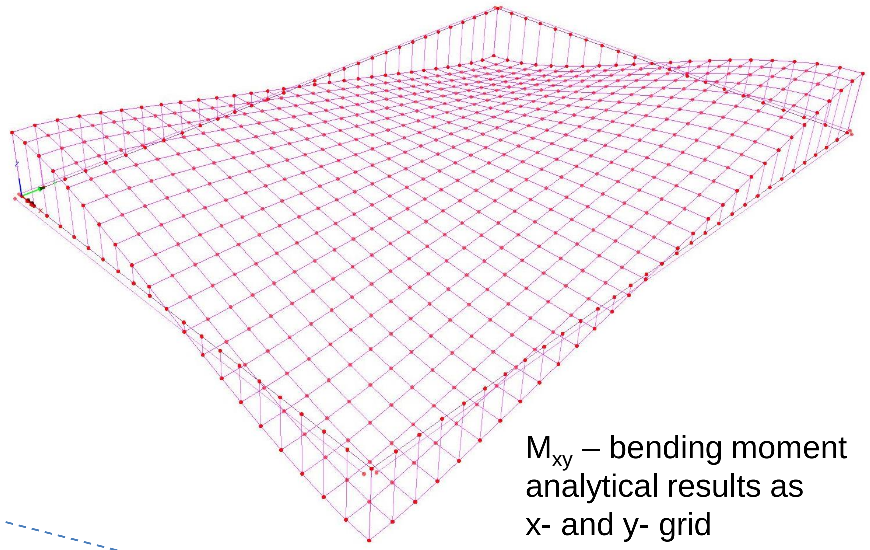
Examples: Result components using Navier's solution method

The combined model representation
and result presentations



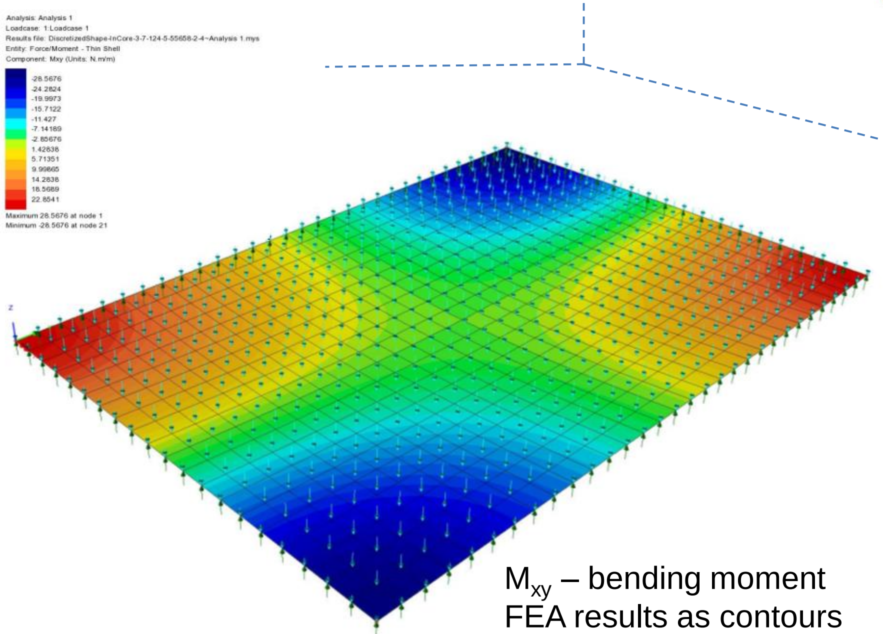
Analytical model

M_{xy} – bending moment
analytical result as
contours



M_{xy} – bending moment
analytical results as
x- and y- grid

FEA model



M_{xy} – bending moment
FEA results as contours

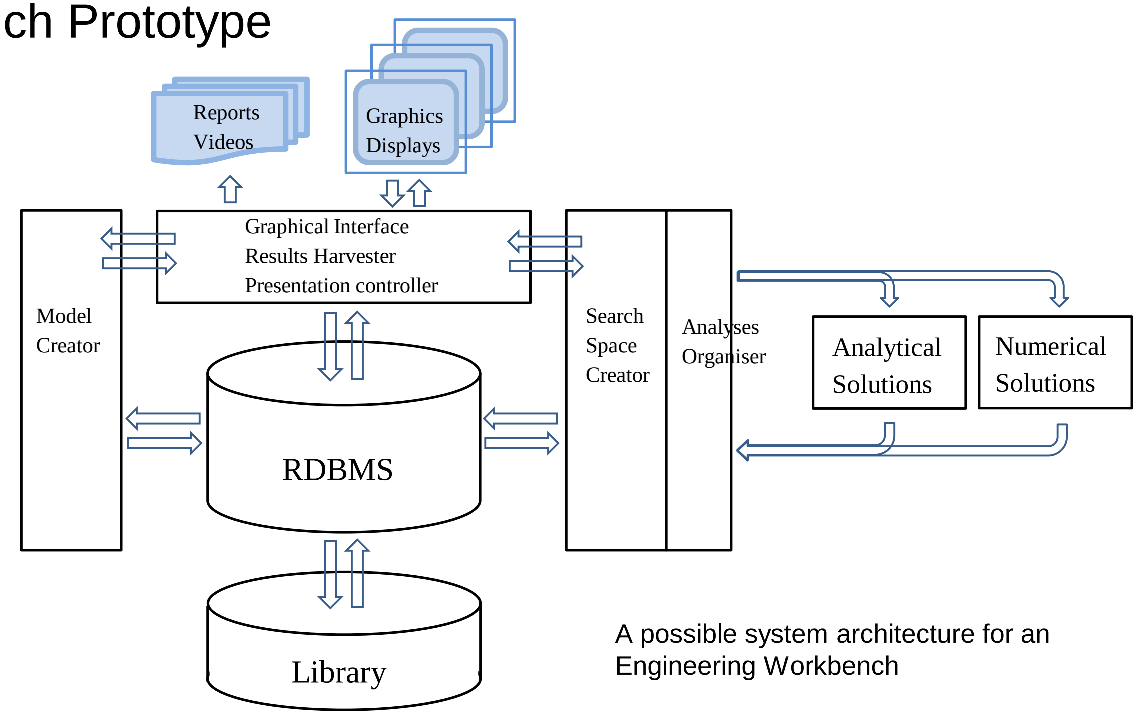
M_{xy} – bending moment
FEA results as x- and y- grid

The analytical results are presented as x- and y- grid
The FEA results are presented as contours

The complementary combinations are possible and
under development

An Engineering Workbench Prototype

My "Old to New" project:



A possible system architecture for an Engineering Workbench

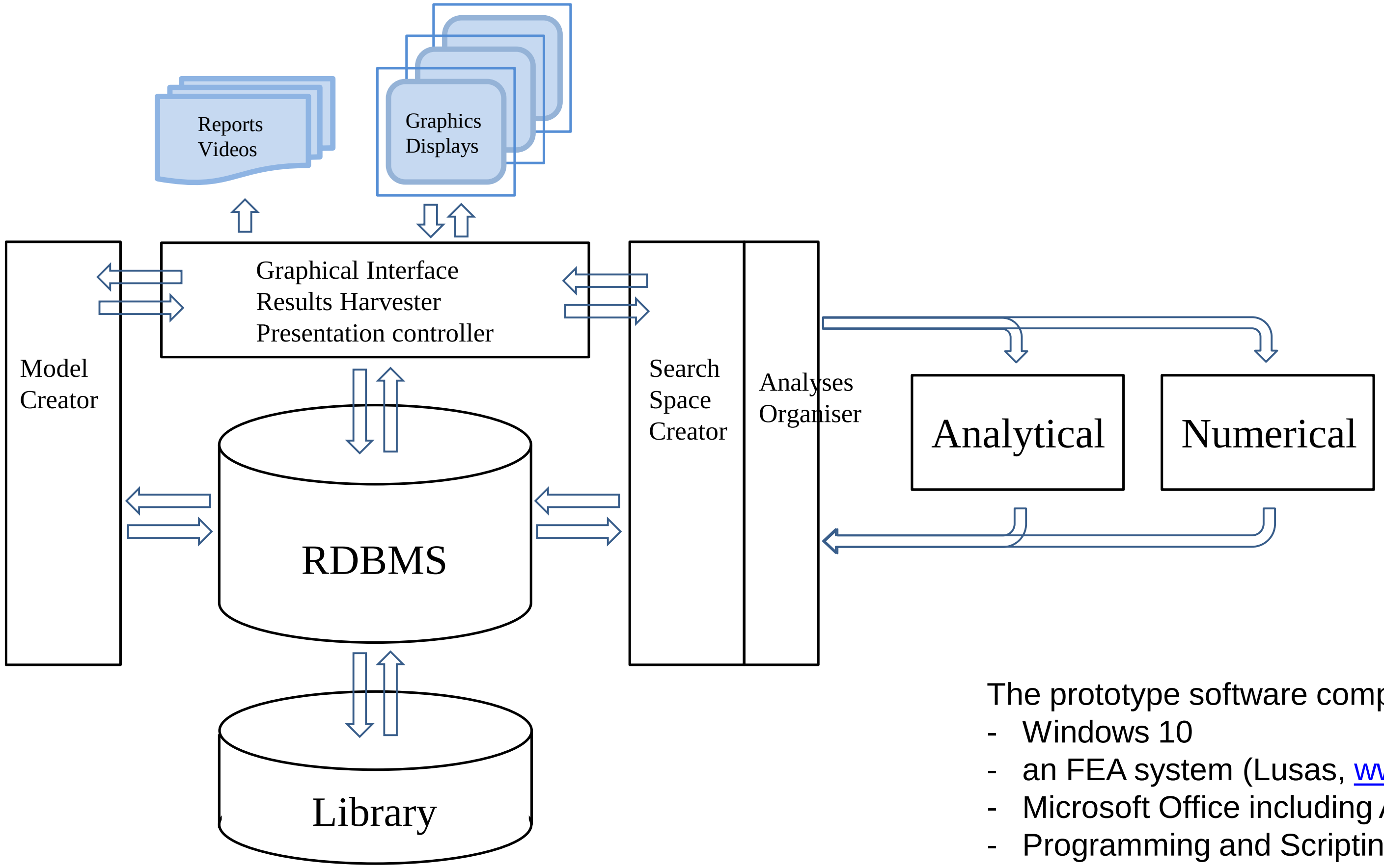
As a young engineer I realised that I had to solve a few problems that blocked the path to where I wanted to go: to develop an Engineering Workbench where FEA analyses could be run effectively using the best technology available for interactive studies of "change". The technology blockers I have worked on throughout my career include:

- Replacing batch oriented use of mainframes with interactive use of mini-machines, work stations, etc.
- Replacing the interface between CAD and FEA with integration
- Solve the problem of Automatic Hexahedral Meshing

In the process, I have been given the custody of ideas on a general mathematical theory for shape representation and manipulation in computer systems. One application is Automatic Hexahedral Meshing, others include the development of a mathematical continuum from parametric CAD to FEA results representation and back, solving the "Island of Automation" conundrum once and for all, and much, much more besides … (attend my other presentation at NWC19).

When the Old meets the New …

An Engineering Workbench Prototype



The prototype software components,
- Windows 10
- an FEA system (Lusas, www.lusas.com)
- Microsoft Office including Access, PowerPoint
- Programming and Scripting languages

The prototype makes use of well-established software components found on the desk of any designer and analyst throughout the industry.
Of particular importance is the RDBMS system Access (soon interfaced to and complemented with SQL Server).
All the software modules are highly programmable through programming and scripting languages like VBA and VBS.

When the Old meets the New …

An Engineering Workbench Prototype

The Old

Theory on PDEs solution methods some equations	Tables for a variety of parameter values	Practical problems with real world dimensions. loads and boundary conditions

A huge gap, too wide for practicing engineers in a hurry

The New

Theory on many PDEs, comprehensive solution methods and all result components	database application for solving various PDEs beyond CPT	Practical problems with real world dimensions, loads and boundary conditions

The gap filled with an office automation application for practicing engineers

It is important to lower the abstraction level in all presentation formats, so a user can relate the analytical and numerical results directly to their own problems.
There should be no tables to extract data from for additional manual calculations.

An Engineering Workbench Prototype: Requirement Specification

Design of a modern computer system for solving PDEs
The requirement specification

#1: A Correct and complete problem definition

#2: Conversion of the geometry into a discretized model

#3: A comprehensive range of ways of presenting the results

#4: Handling of a range of mathematical models

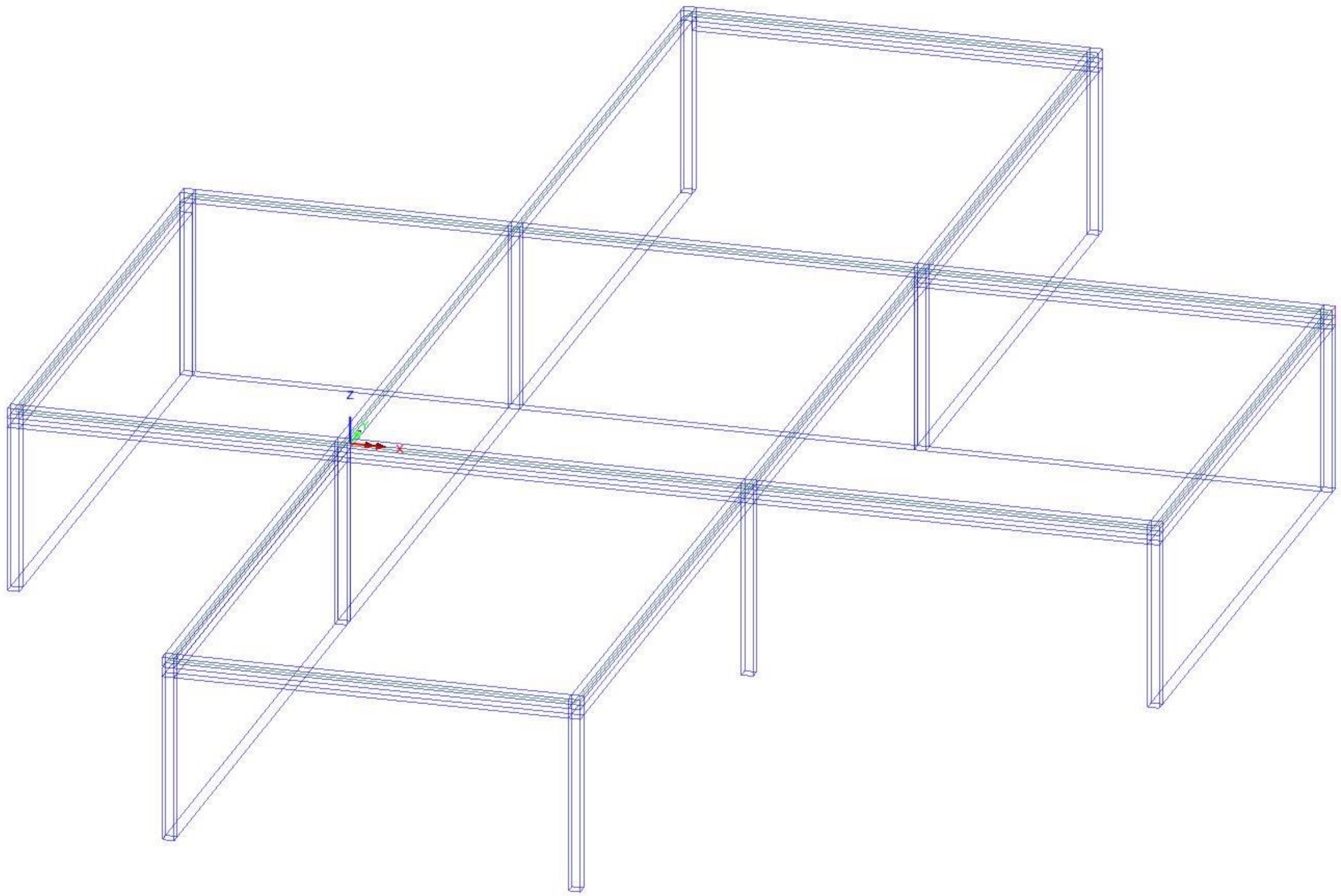
#5: A multi-dimensional search space

#6: A modern user interface

The implementation is progressing at pace, this presentation is a progress report.

A Correct and Complete Problem Definition: A practical problem

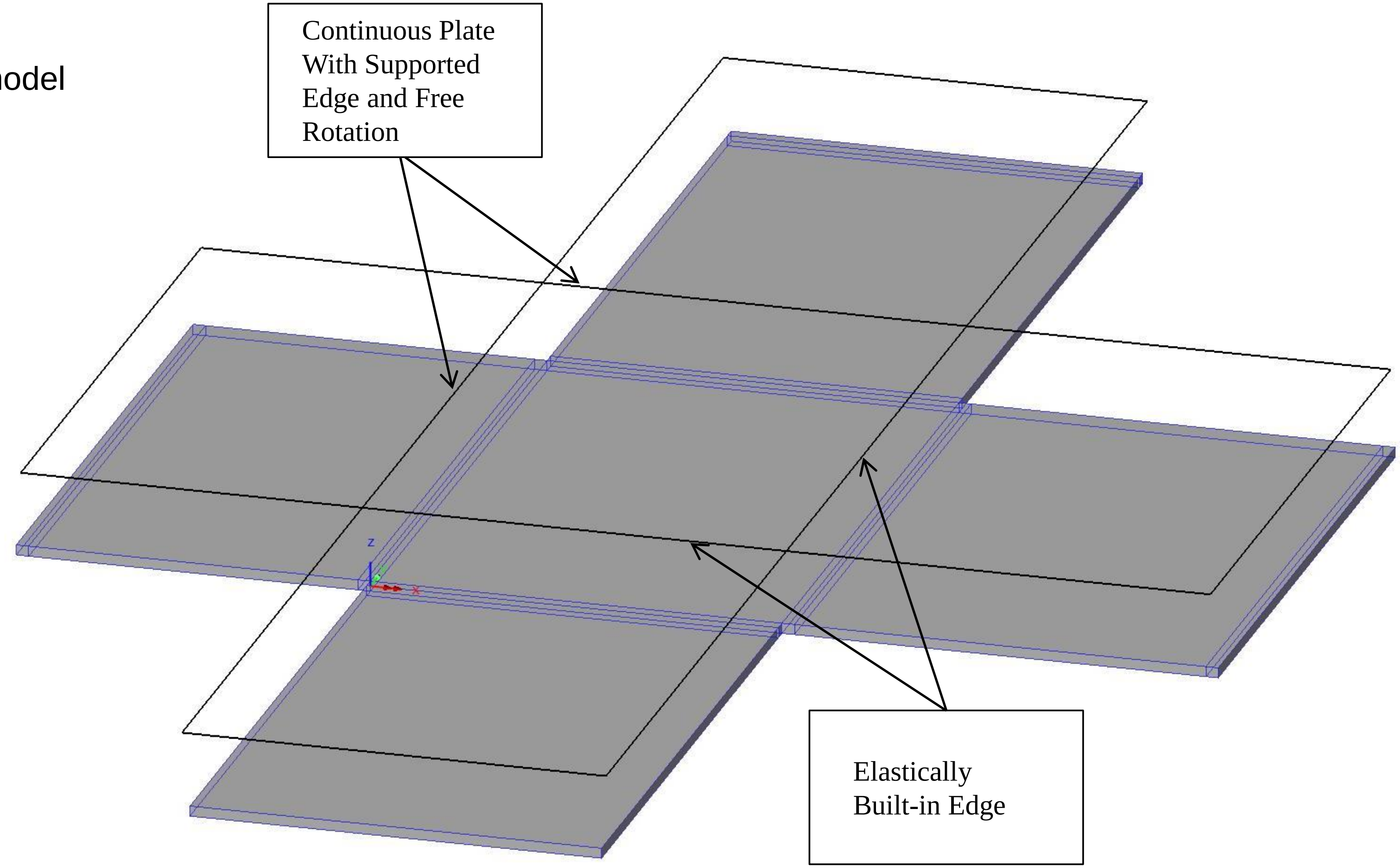
The real shape



A practical problem often looks very different to the theory in the books. Here is completely made up example, one that should illustrate the nature of real engineering problems. A collection of continuous plates join the centre plate from four sides where the centre plate rests on a ridgid wall and flexible beams alternatively. The task is to calculate the deformations and bending moment distributions for subsequent Serviceability Limit State (SLS) and Ultimate Limit State (ULS) calculations for the plate assembly, focusing on the centre plate.

A Correct and Complete Problem Definition: A practical problem

The idealised model



The idealised model must be represented so loads and boundary conditions reflect the physical shape and behaviour of the structure.
The boundary conditions for a continuous plate on top of a wall can be modelled as shown above.
However, the other two edges rest on top of a elastic beam and should include the deformation of the beam in the boundary condition.
The nearest we get with the boundary conditions in our list is the Elastically Built-in Edge.

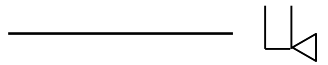
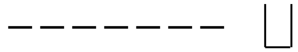
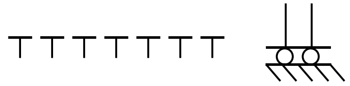
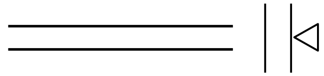
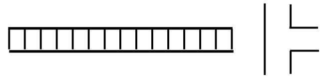
When the Old meets the New …

A Correct and Complete Problem Definition: Boundary Conditions

Taken from “Ernst Bittner: Platten und Behälter”,

“In the books” (Simply Supported Edge, Fixed or Built-in Edge, Free Edge)

“In practical problems” and in one of the books … (Sliding Edge With Fixed Rotation, Continuous Plate With Supported Edge and Free Rotation, Elastically Built-in Edge)

Boundary Condition Name	Mathematical Expression			Graphical Symbols
Simply Supported Edge	$w = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$	for x=0 and x=a	
	$w = 0$	$\frac{\partial^2 w}{\partial y^2} = 0$	for y=0 and y=b	
Fixed or Built-in Edge	$w = 0$	$\frac{\partial w}{\partial x} = 0$	for x=0 and x=a	
	$w = 0$	$\frac{\partial w}{\partial y} = 0$	for y=0 and y=b	
Free Edge	$\frac{\partial^2 w}{\partial x^2} = 0$	$[\frac{\partial^3 w}{\partial x^3} + (2\text{-}\nu)\frac{\partial^3 w}{\partial x \partial y^2}] = 0$	for x=0 and x=a	
	$\frac{\partial^2 w}{\partial y^2} = 0$	$[\frac{\partial^3 w}{\partial y^3} + (2\text{-}\nu)\frac{\partial^3 w}{\partial x^2 \partial y}] = 0$	for y=0 and y=b	
Sliding Edge With Fixed Rotation	$\frac{\partial w}{\partial x} = 0$	$[\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}] = 0$	for x=0 and x=a	
	$\frac{\partial w}{\partial y} = 0$	$[\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y}] = 0$	for y=0 and y=b	
Continuous Plate With Supported Edge and Free Rotation	$w = 0$	$\frac{\partial w}{\partial xleft} = \frac{\partial w}{\partial y_{right}}$	for x=0 and x=a	
	$w = 0$	$\frac{\partial w}{\partial y_{left}} = \frac{\partial w}{\partial y_{right}}$	for y=0 and y=b	
Elastically Built-in Edge	$w = 0$	$\frac{\partial w}{\partial xplate} = \frac{\partial w}{\partial xsupport}$	for x=0 and x=a	
	$w = 0$	$\frac{\partial w}{\partial y_{plate}} = \frac{\partial w}{\partial y_{support}}$	for y=0 and y=b	

Note: all the 10 Result Components listed earlier are represented as boundary conditions in the table M_{xy} only indirectly in V_x and V_y).

There are more boundary conditions than those included in the table.

For example, a Simply Supported Edge may be free to roll forwards, and the u and v displacements may be active when the PDE cater for them. If these displacements are free, the plate moves freely, if they are fixed, membrane forces may build up and stiffen the plate.
So, what is it?

For example, the Elastically Built-in Edge may rest on a flexible beam, and w = 0 must be replaced with the deflection along the edge matching the beam theory that applies.

A Correct and Complete Problem Definition: Boundary Conditions

The number of alternative boundary conditions can be extended beyond the table in the previous slide.
Their usefulness start beyond the Navier's Solution Method and can be modelled using Levy's Method and the other approximate analytical methods. Three criteria apply:

- There is a physical boundary conditions that need to be modelled correctly;
- The PDE can represent the derivative representation of the expression;
- The boundary condition can be modelled in FEA terms.

The best practical book I have come across is "Ernst Bittner: Platten und Behälter", Springer Verlag, 1965, where he uses Levy's solution method to solve the CPT and to enumerate all sorts of permutations for boundary conditions.

- There is a list of boundary conditions that are considered (the table in the next slide is taken from Ernst Bittner;
- To allow for additional boundary conditions beyond simply supported on the two adjacent edges where x=0 and x=a, he advocates the use of a bending moment on the edge to added to the first solution.

- There is a long list of load types that are included in the practical use of the theory

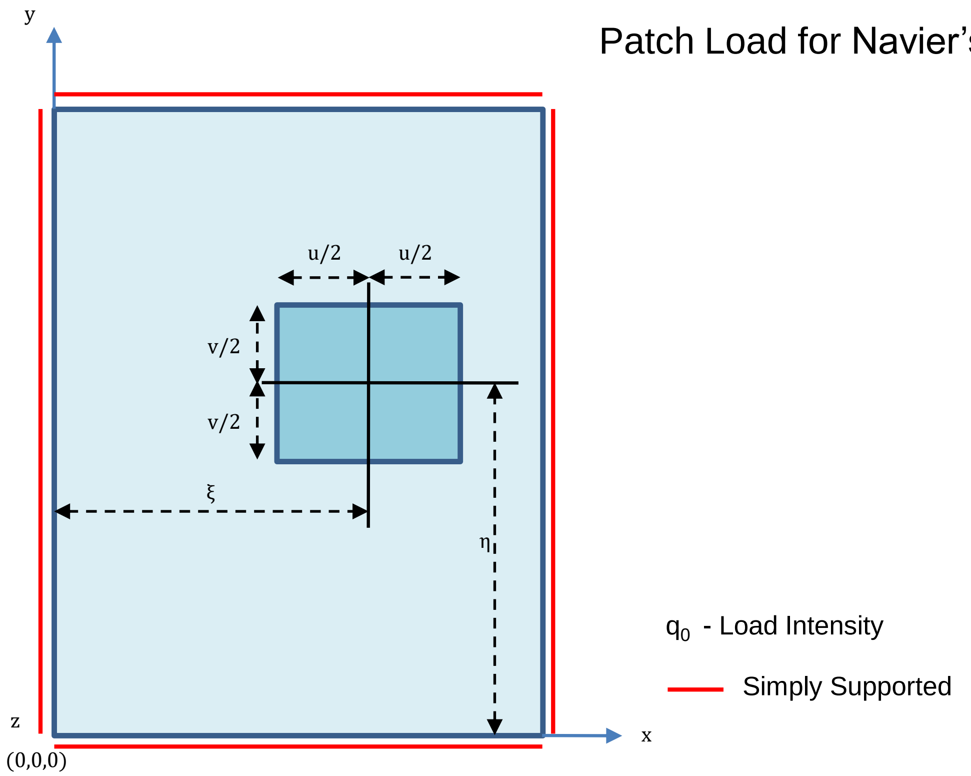
All this ends up in a large collection a specific displacement expressions and its derivatives ready to be enumerated in a number of tables.

He subsequently goes through a number of solution examples, showing how to make use of the formulae presented to solve practical problems.

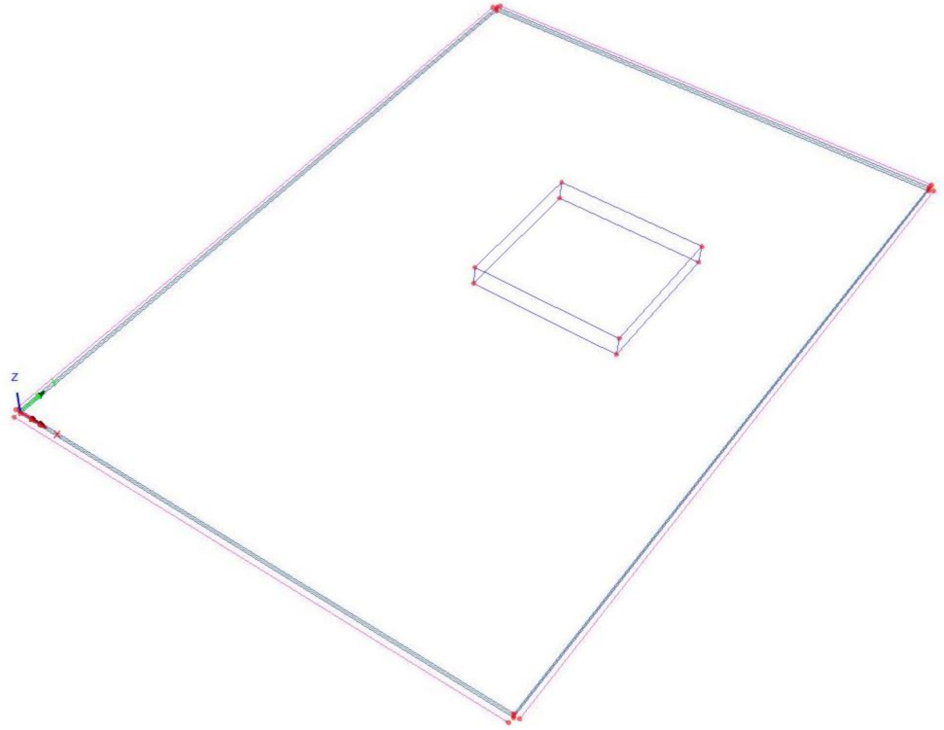
The bulk of the book was developed long before computers became available.

He even consistently uses a righthand coordinate system in the bottom left-hand corner of the plate.

A Correct and Complete Problem Definition: Patch Load Example



Patch Load for Navier's solution method



$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

$$a_{mn} = \frac{16 q_0}{\pi^2 m n u v} Sin \frac{m\pi\xi}{a} Sin \frac{n\pi\eta}{b} \; Sin \frac{m\pi u}{2a} \; Sin \frac{n\pi v}{2b}$$

Patch load anywhere with Levy's solution method:
A patch load like this can be represented using Navier's double Fourier series solution method. T&WK demonstrates a solution using Levy's single Fourier series in X, but the load has a fixed position.
We need a single Fourier series solution (in X and Y) where the right-hand coordinate system is in the lower left-hand corner of the plate and the patch load can move anywhere within the plate boundary. Anyone interested in developing the Levy solutions?

A Correct and Complete Problem Definition: Other Load Types

a_{mn} for alternative load types for Navier's solution method

Uniformly distributed load
$$a_{mn} = \frac{16qo}{\pi^2 mn} \qquad (m,n = 1,3,5,\ldots)$$

Uniformly distributed rectangular patch load
$$a_{mn} = \frac{16qo}{\pi^2 mnuv} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \sin\frac{m\pi u}{2a} \sin\frac{n\pi v}{2b} \qquad (m,n = 1,3,5,\ldots)$$

Hydrostatic load
$$a_{mn} = \frac{8q_o}{\pi^2 mn}(-1)^{n+1} \qquad (m = 1,3,5,\ldots)\ (n = 1,2,3,\ldots)$$

Point load
$$a_{mn} = \frac{4q_o}{ab} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \qquad (m,n = 1,3,5,\ldots)$$

Line load $f(x,y) = q_o(y-\eta)$
$$a_{mn} = \frac{8q_o}{\pi bm} \sin\frac{n\pi\eta}{b} \qquad (m = 1,3,5,\ldots)\ (n = 1,2,3,\ldots)$$

Line load $f(x,y) = q_o(x-\xi)$
$$a_{mn} = \frac{8q_o}{\pi an} \sin\frac{m\pi\xi}{a} \qquad (m = 1,3,5,\ldots)\ (n = 1,2,3,\ldots)$$

Uniformly distributed circular load
$$a_{mn} = \frac{8q_o}{abc\gamma_{mn}} J_1(\gamma_{mn}c) \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \qquad (m,n = 1,3,5,\ldots)$$

$$\gamma_{mn} = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

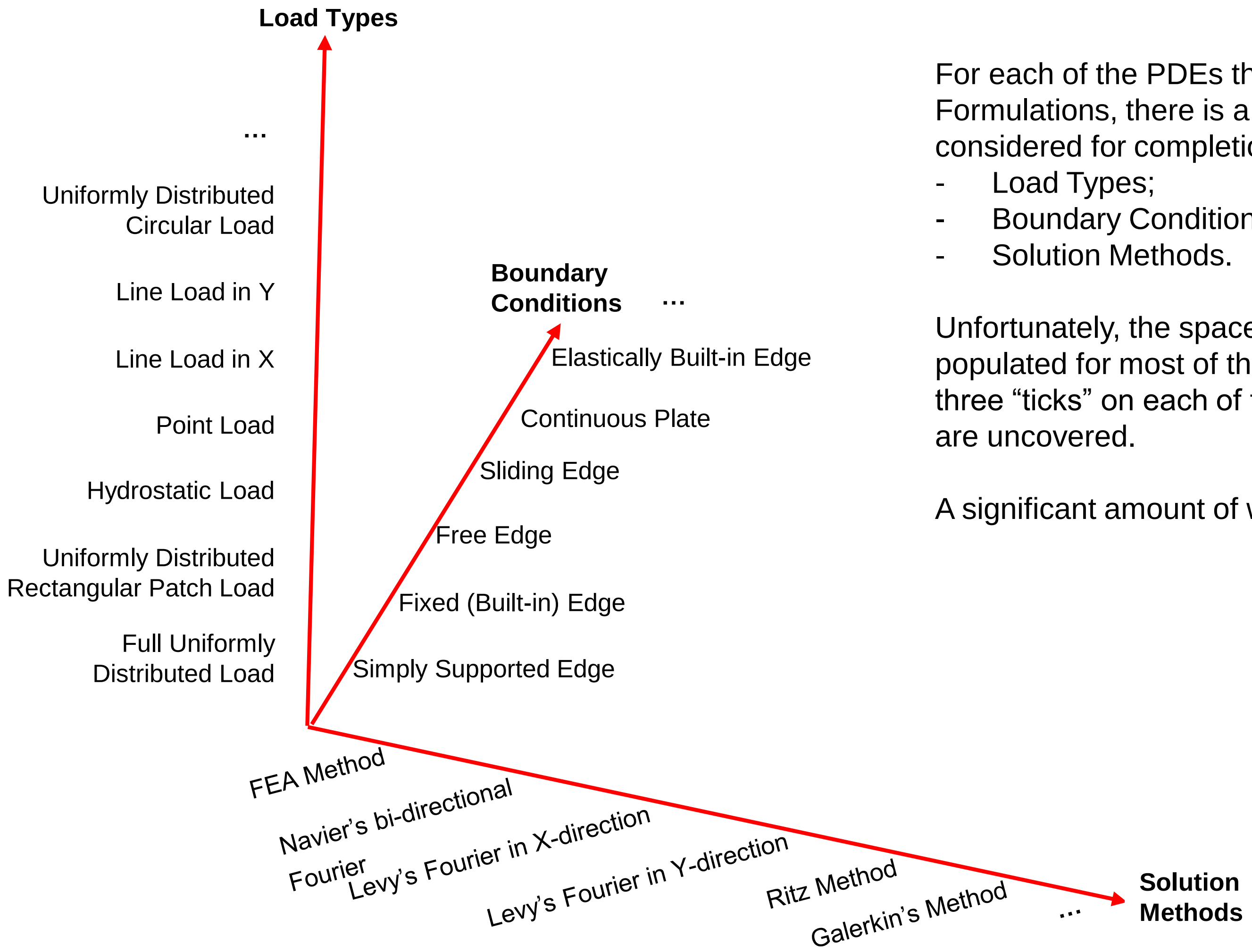
c is the radius of the circle with its centre in $x = \xi$ and $y = \eta$

$J_1(\gamma_{mn}c)$ is the Bessel function of order one, with the argument $\gamma_{mn}c$

All a_{mn} functions taken from T&WK and there are more, there and elsewhere.

When the Old meets the New …

A Correct and Complete Problem Definition: a PDE definition space



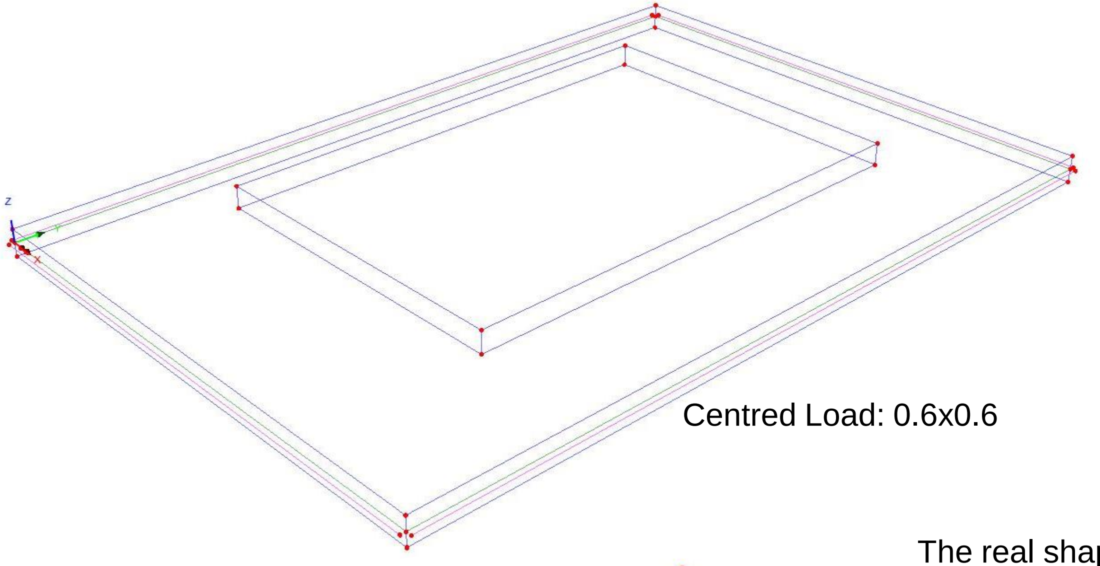
For each of the PDEs that can be developed using the alternative Formulations, there is a space of additional parameters that all need to be considered for completion:

- Load Types;
- Boundary Conditions;
- Solution Methods.

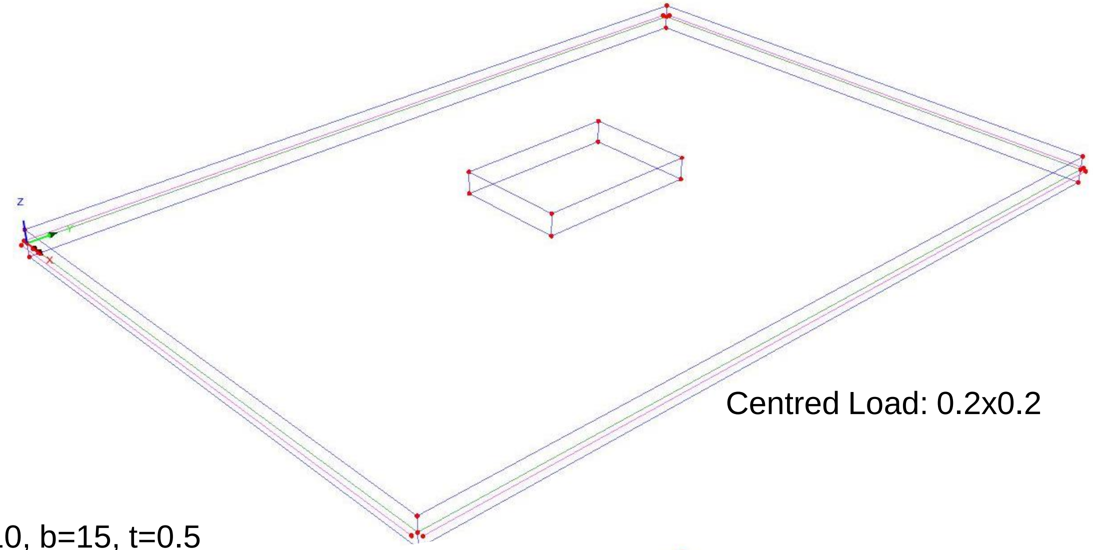
Unfortunately, the space spanned by these parameters are sparsely populated for most of the PDEs. Effort has been focused on the first three "ticks" on each of the three axes, and the other parameter values are uncovered.

A significant amount of work is needed to fill the space with solutions.

A Correct and Complete Problem Definition: Patch Load Example

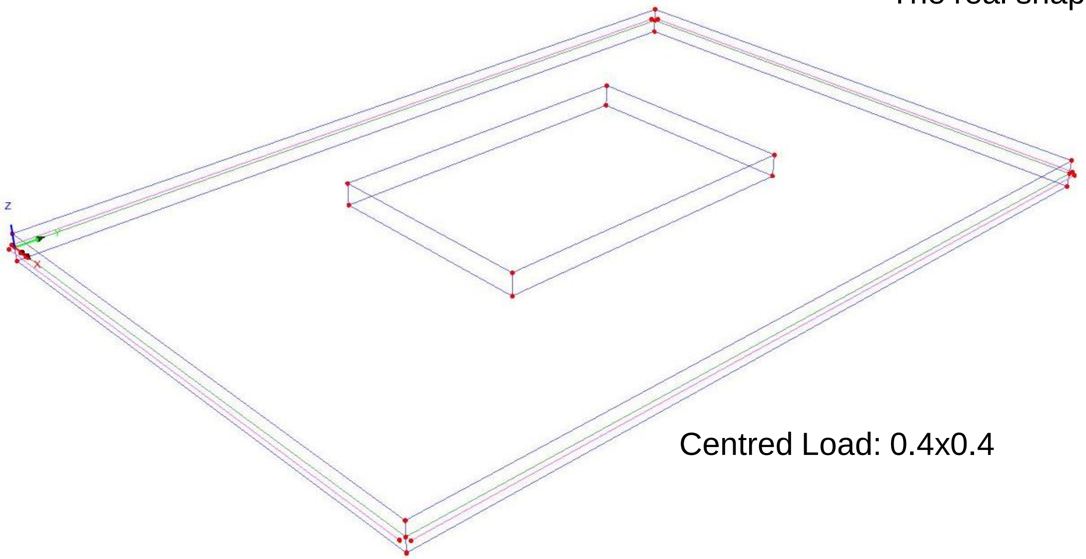


Centred Load: 0.6x0.6

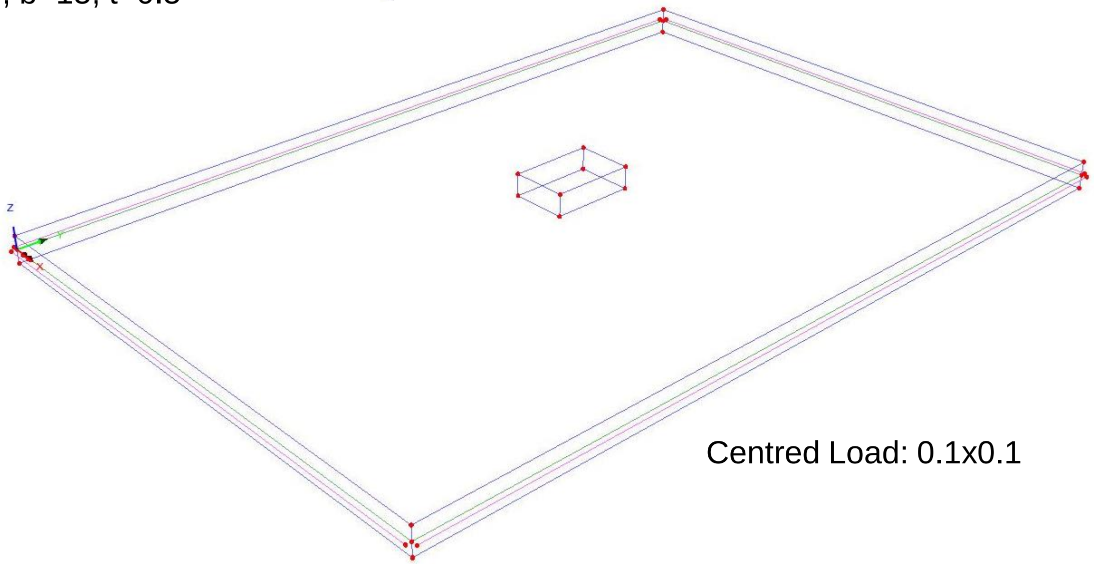


Centred Load: 0.2x0.2

The real shape: a=10, b=15, t=0.5



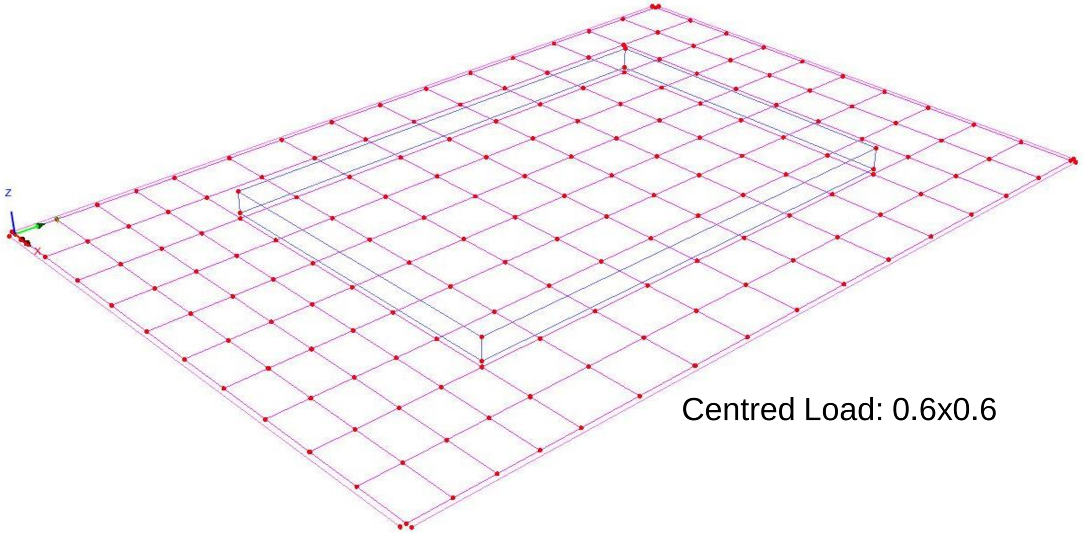
Centred Load: 0.4x0.4



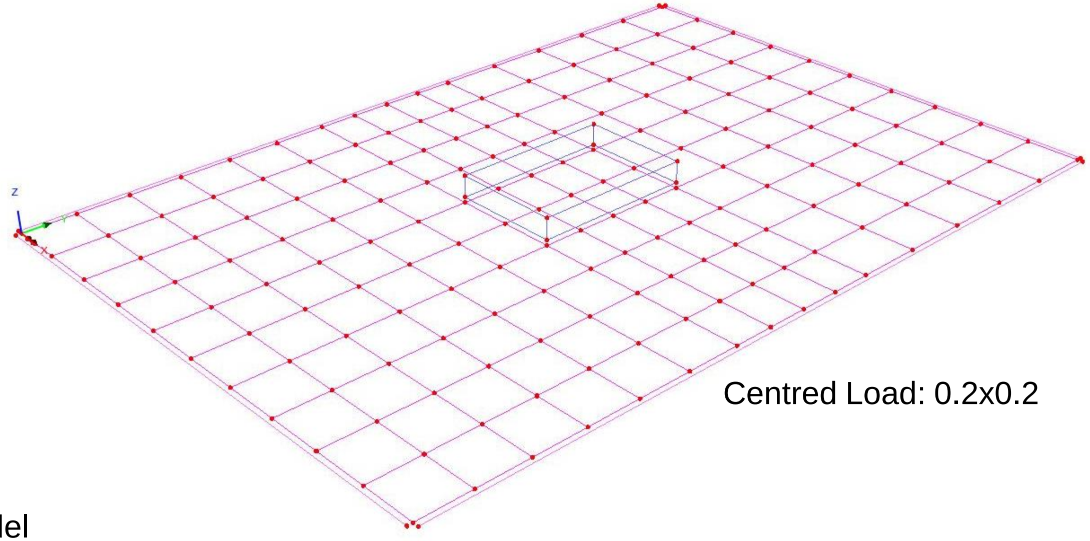
Centred Load: 0.1x0.1

When the Old meets the New …

Conversion of the Geometry into a Discretized Model: Example: Patch Load

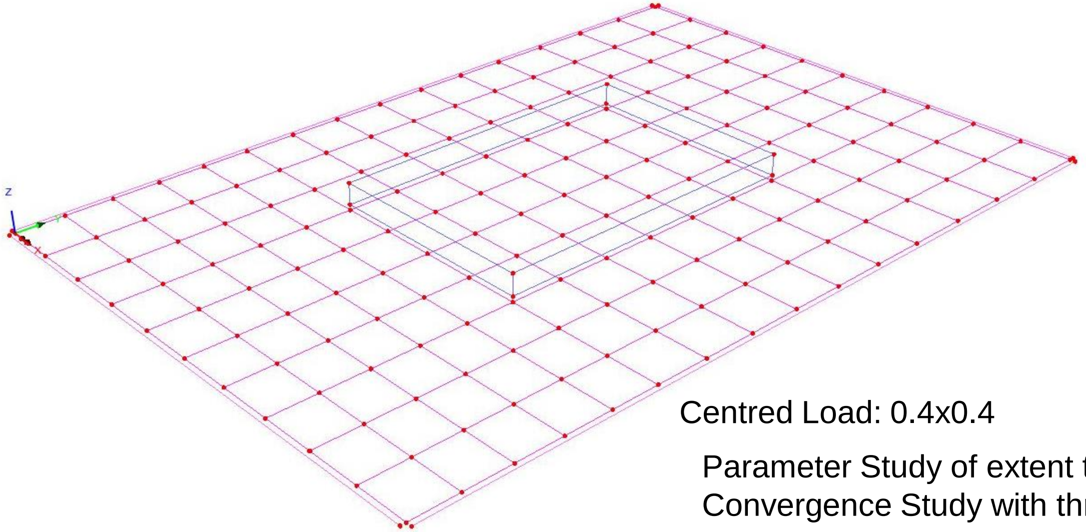


Centred Load: 0.6x0.6



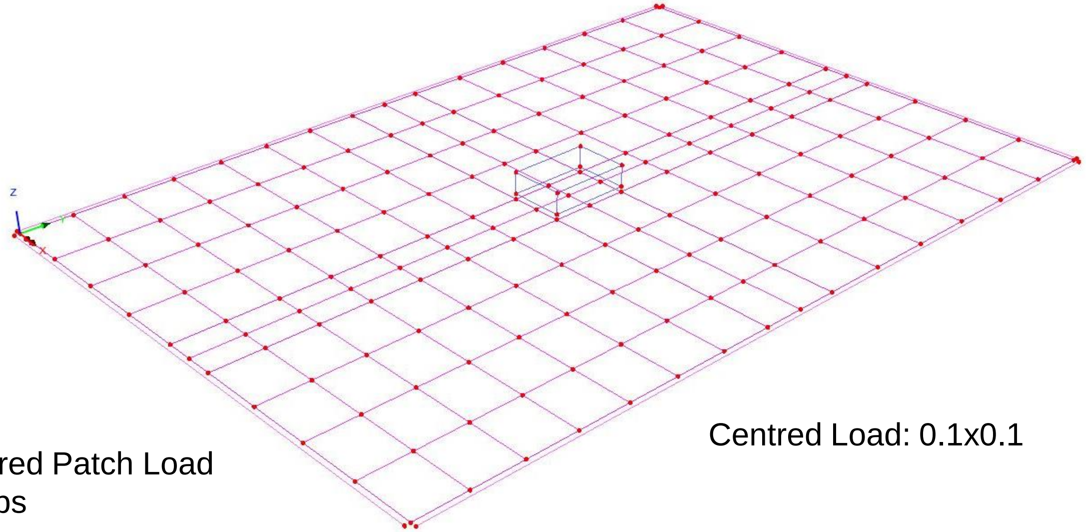
Centred Load: 0.2x0.2

The discretized model



Centred Load: 0.4x0.4

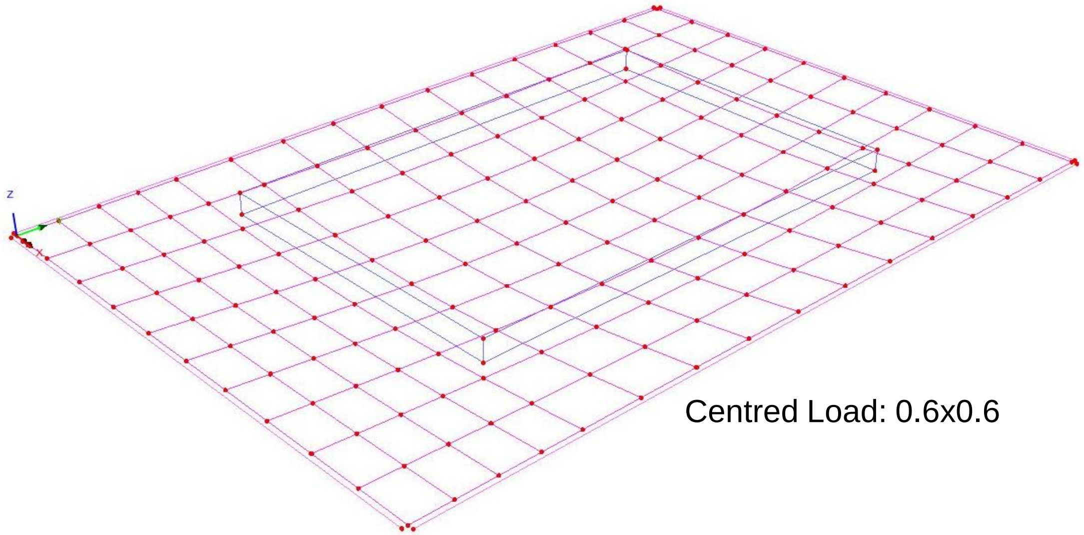
Parameter Study of extent to Centred Patch Load
Convergence Study with three steps



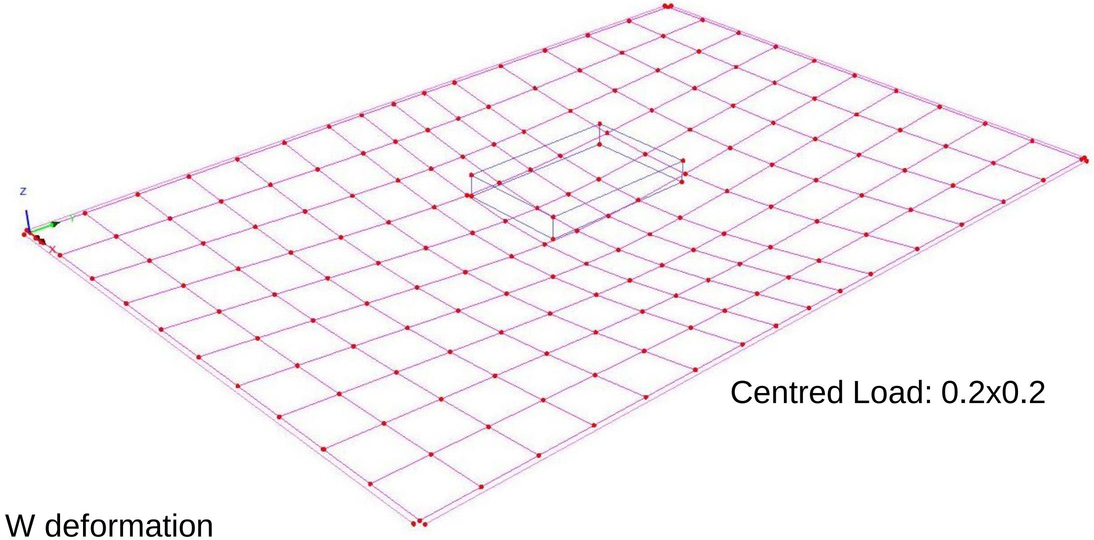
Centred Load: 0.1x0.1

When the Old meets the New …

A comprehensive range of ways of presenting the results: Example: Patch Load

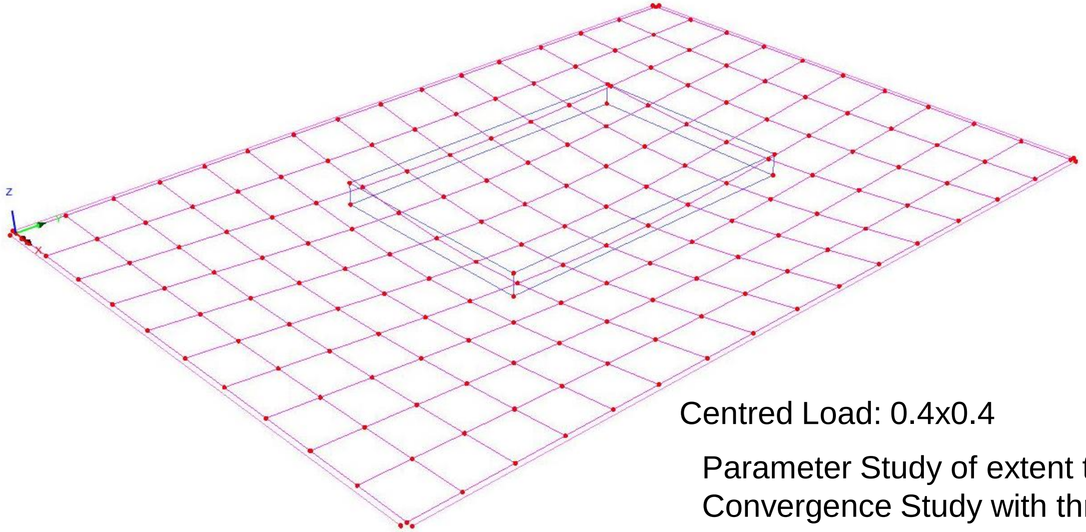


Centred Load: 0.6x0.6

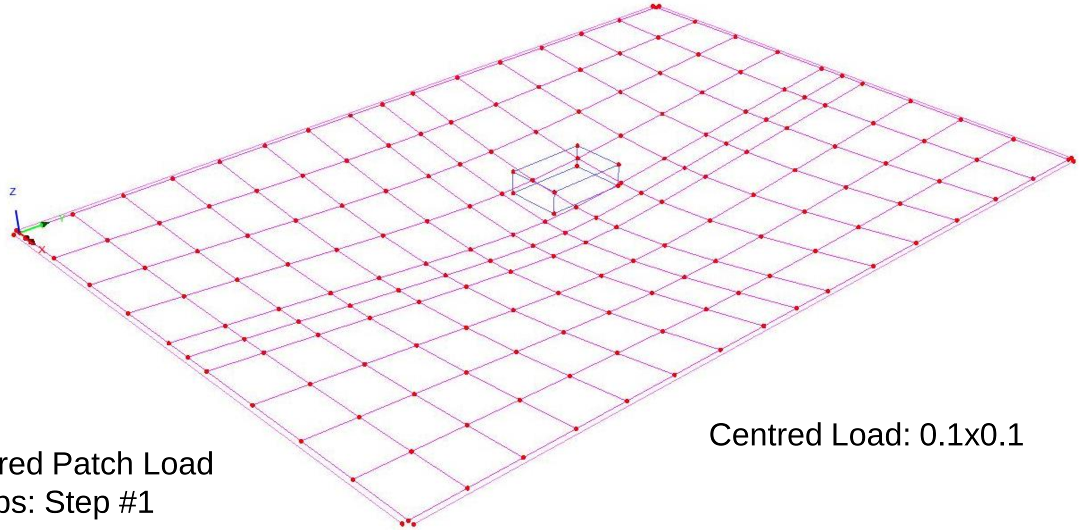


Centred Load: 0.2x0.2

Result Component: W deformation



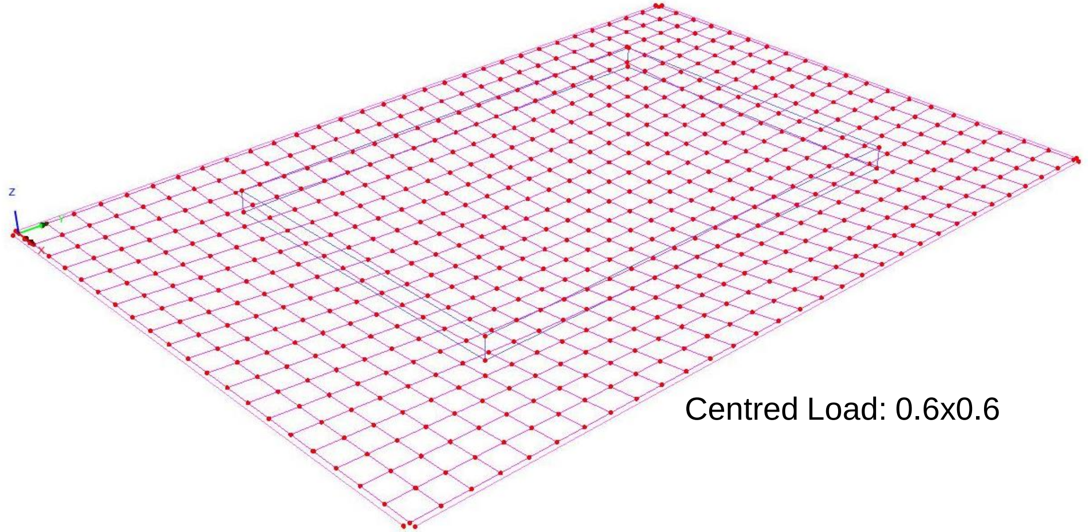
Centred Load: 0.4x0.4



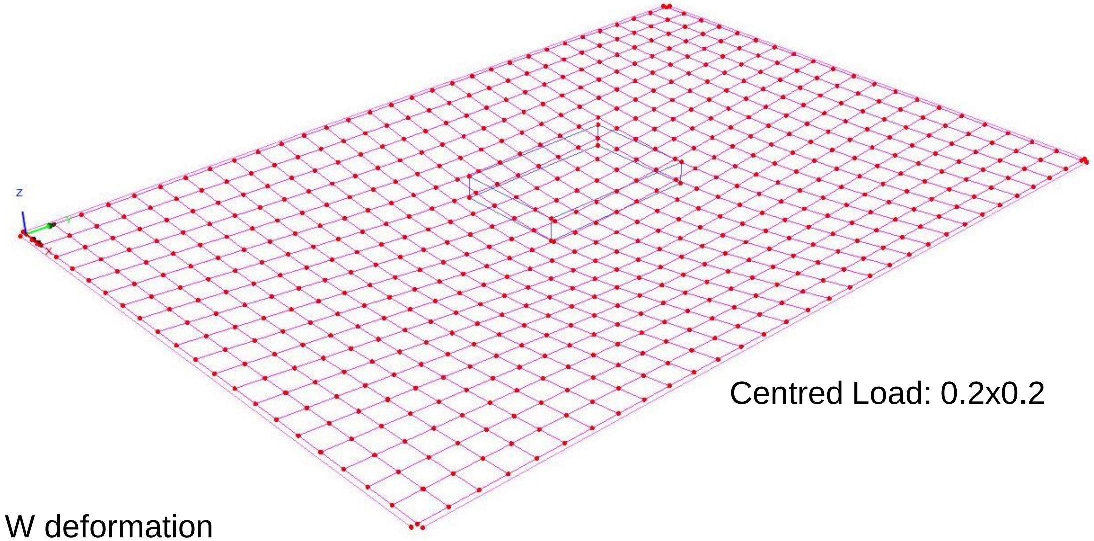
Centred Load: 0.1x0.1

Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #1

A comprehensive range of ways of presenting the results: Example: Patch Load

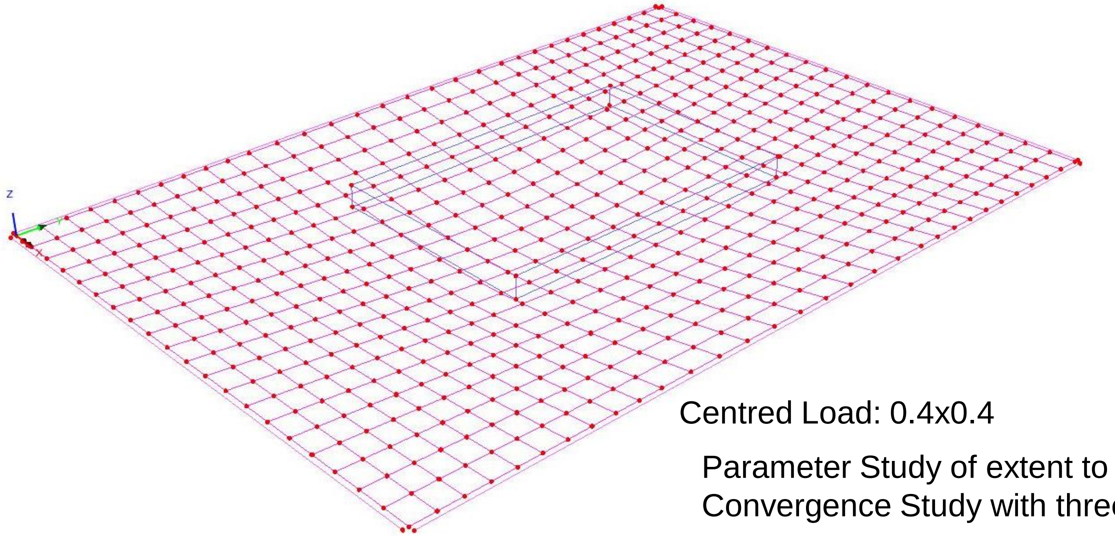


Centred Load: 0.6x0.6

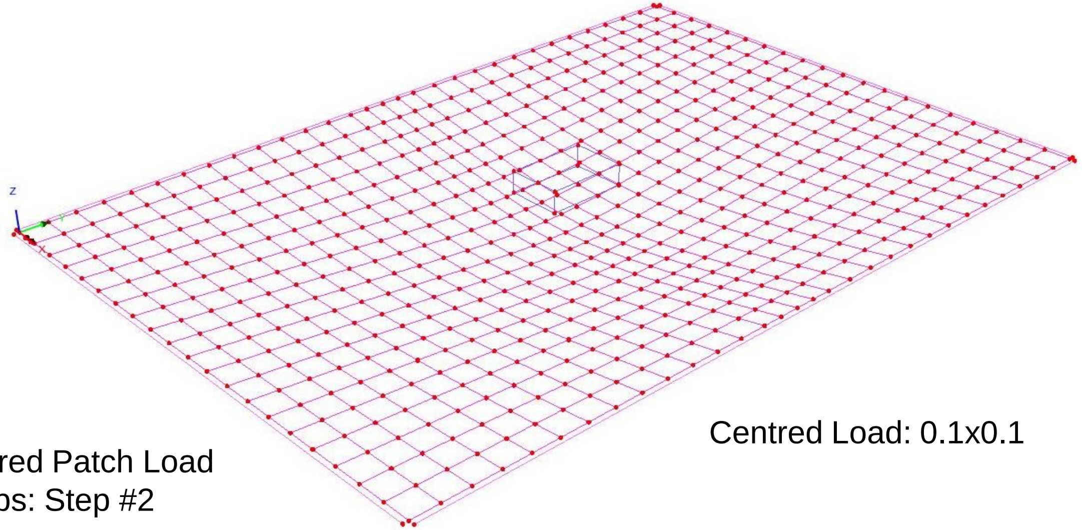


Centred Load: 0.2x0.2

Result Component: W deformation



Centred Load: 0.4x0.4

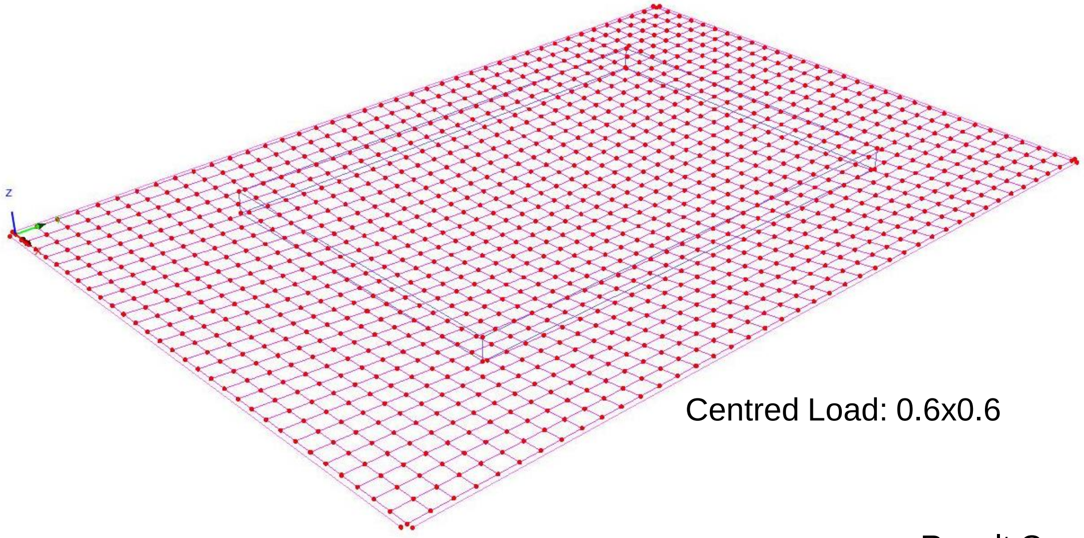


Centred Load: 0.1x0.1

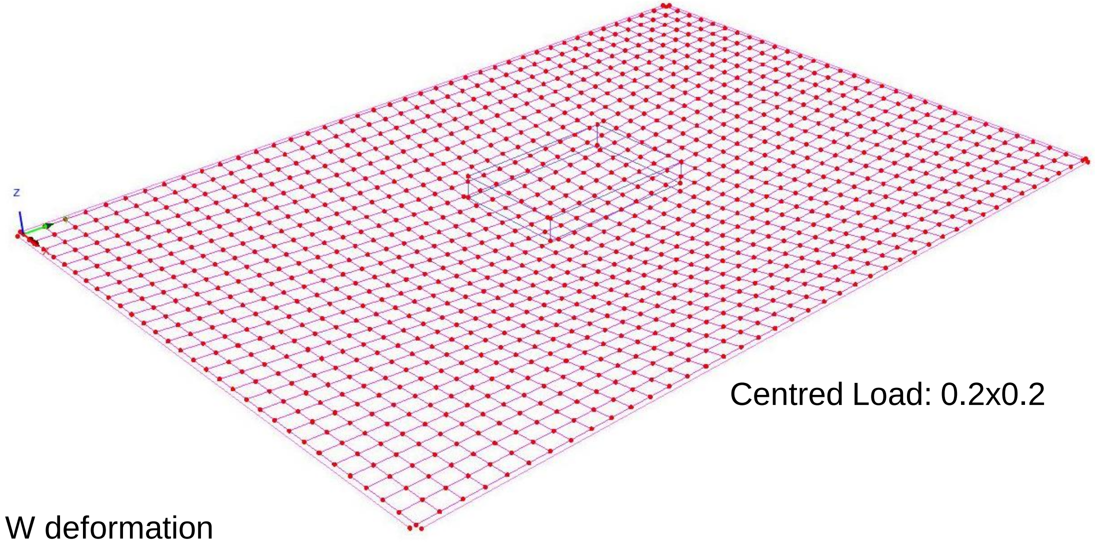
Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #2

When the Old meets the New …

A comprehensive range of ways of presenting the results: Example: Patch Load

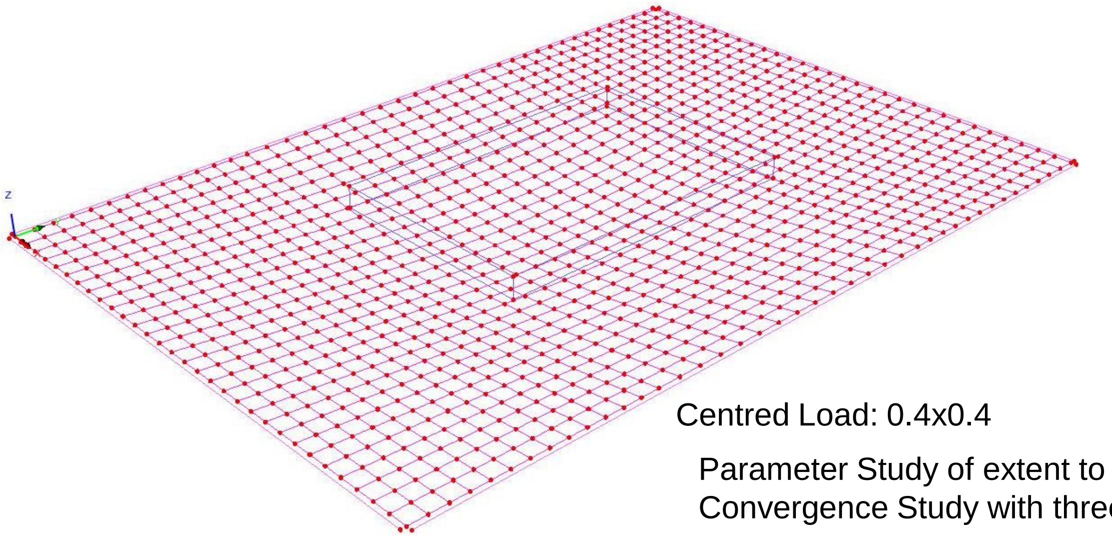


Centred Load: 0.6x0.6

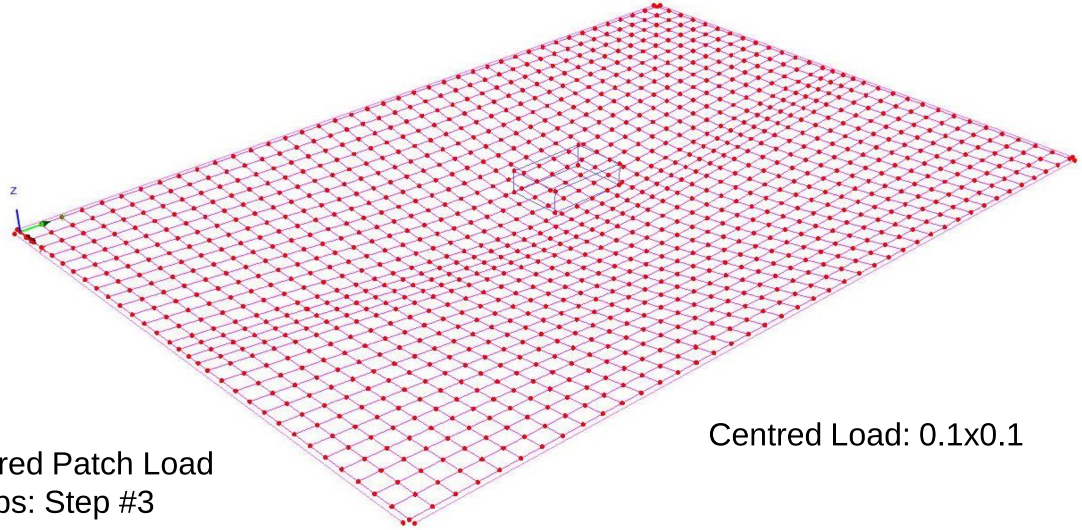


Centred Load: 0.2x0.2

Result Component: W deformation



Centred Load: 0.4x0.4

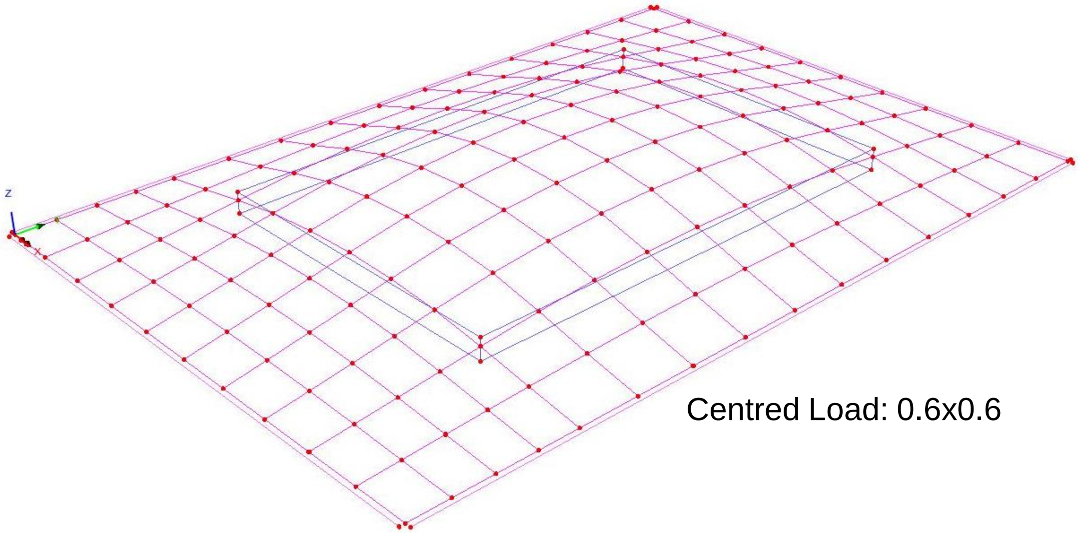


Centred Load: 0.1x0.1

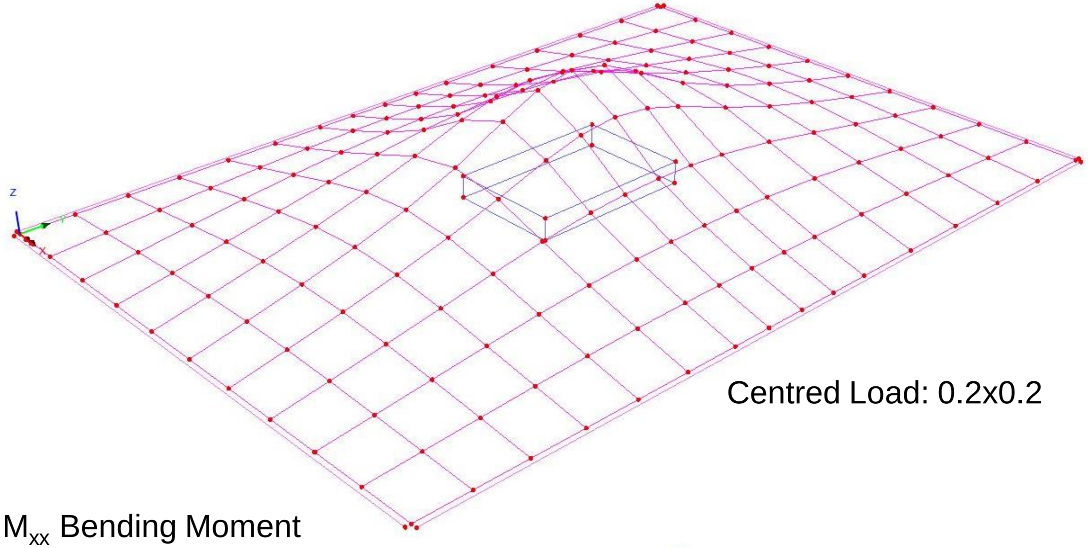
Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #3

The analytical reults: Coarse Mesh

A comprehensive range of ways of presenting the results: Example: Patch Load

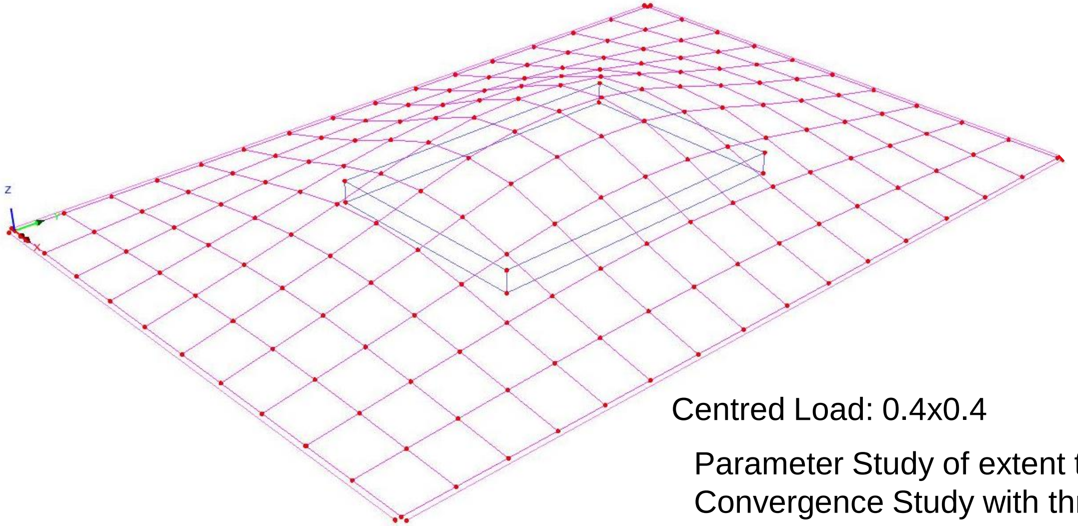


Centred Load: 0.6x0.6

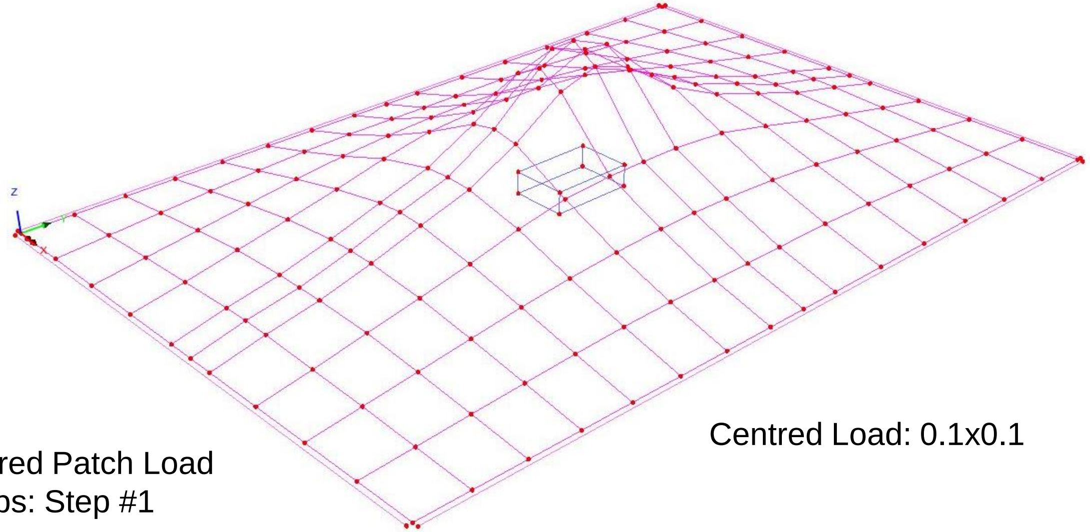


Centred Load: 0.2x0.2

Result Component: M_{xx} Bending Moment



Centred Load: 0.4x0.4

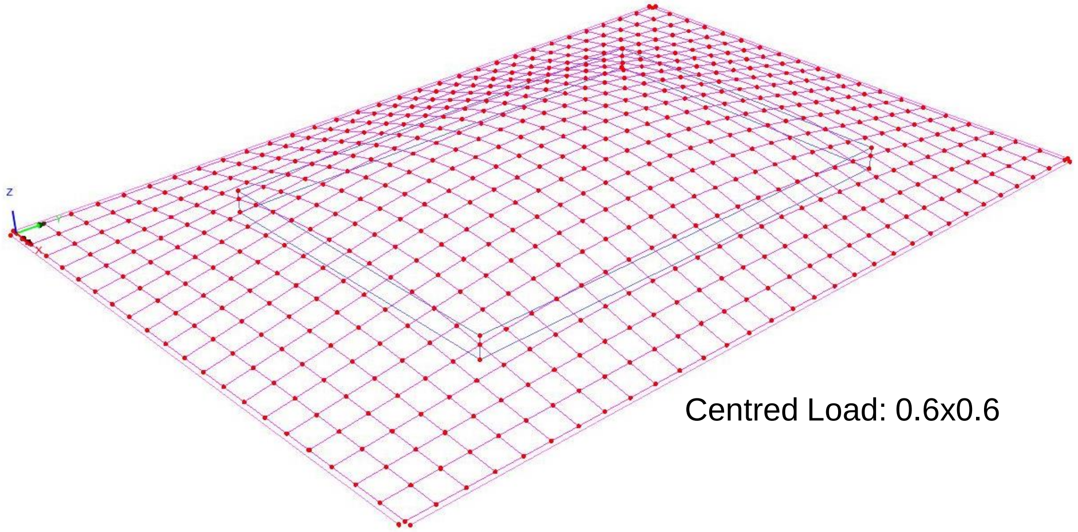


Centred Load: 0.1x0.1

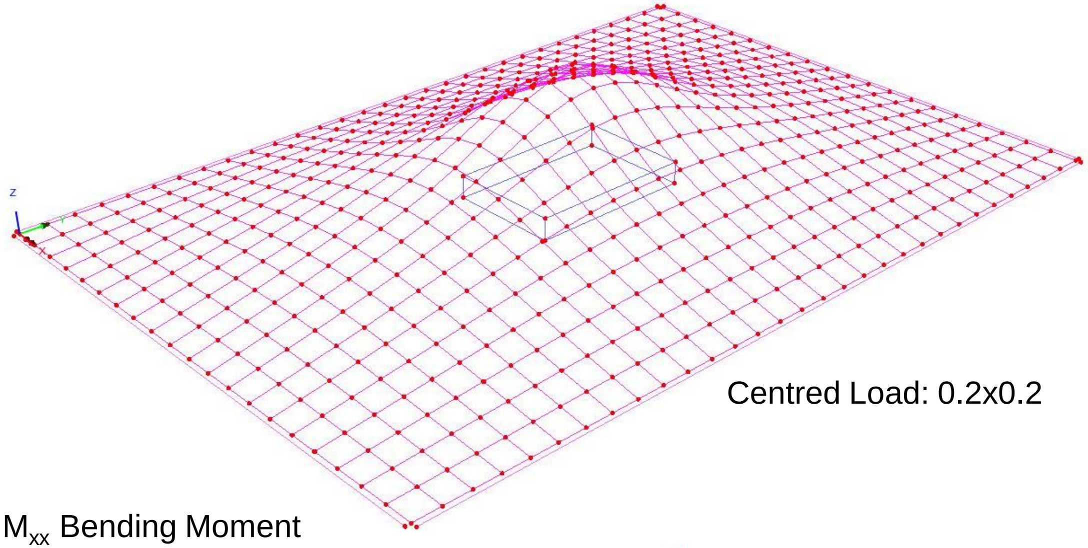
Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #1

When the Old meets the New …

A comprehensive range of ways of presenting the results: Example: Patch Load

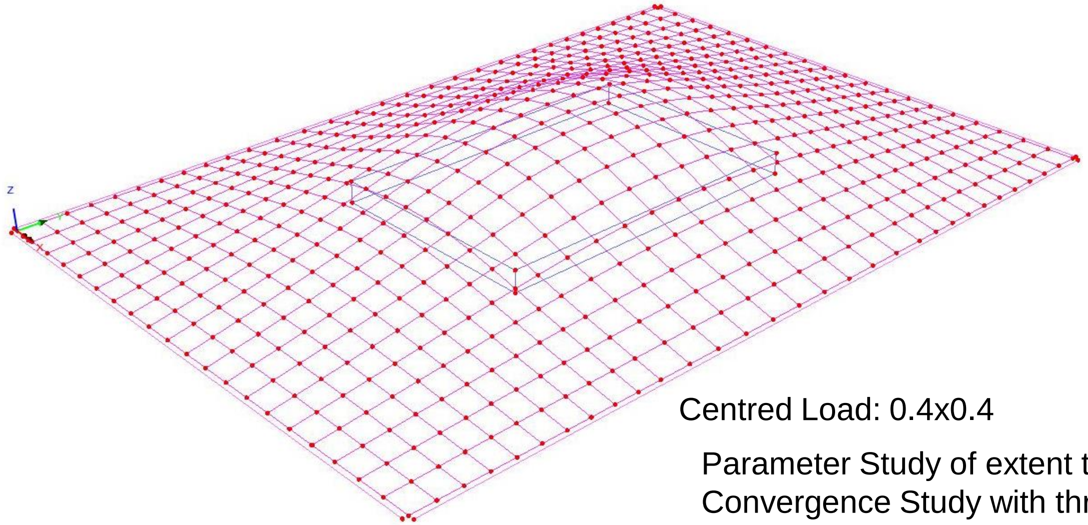


Centred Load: 0.6x0.6

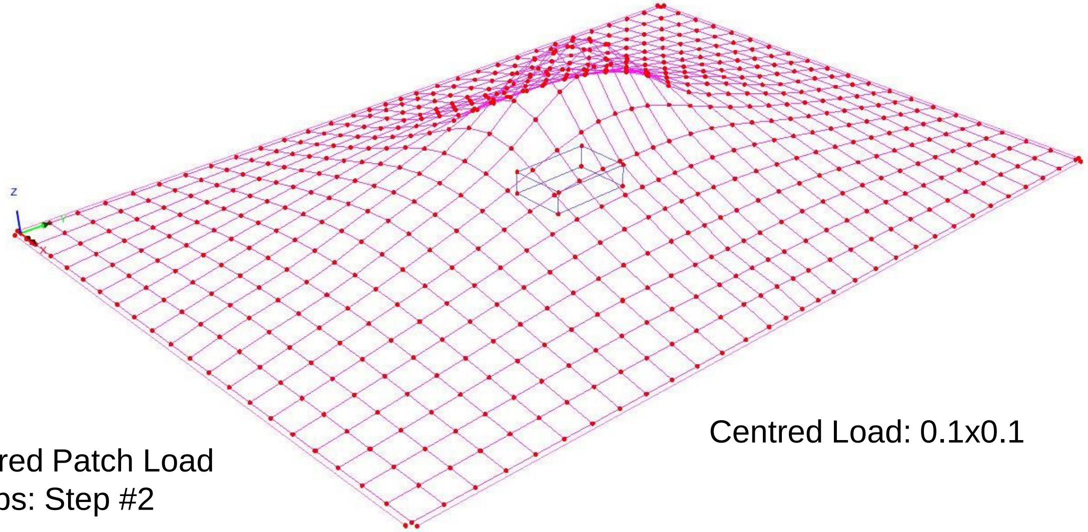


Centred Load: 0.2x0.2

Result Component: M_{xx} Bending Moment



Centred Load: 0.4x0.4

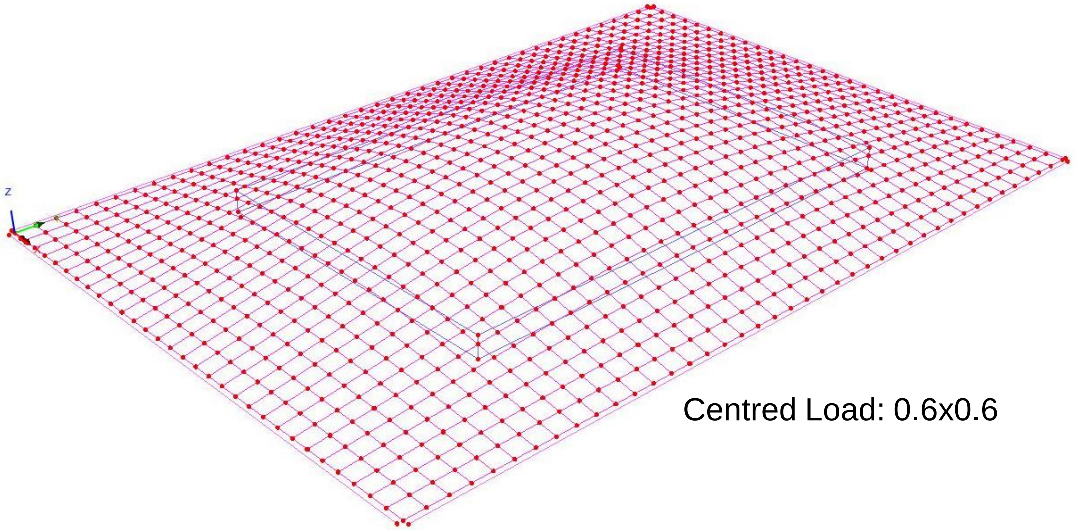


Centred Load: 0.1x0.1

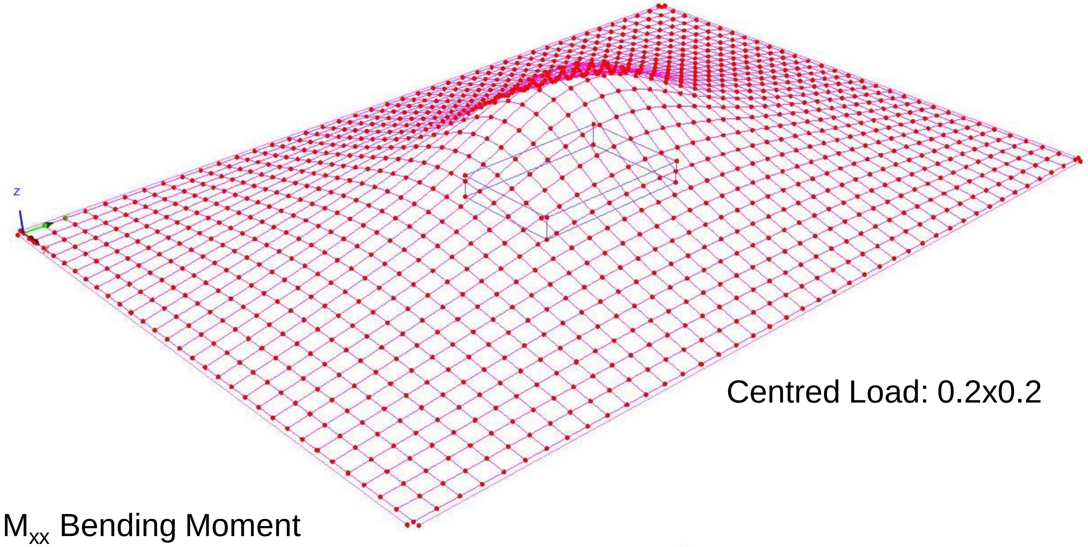
Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #2

When the Old meets the New …

A comprehensive range of ways of presenting the results: Example: Patch Load

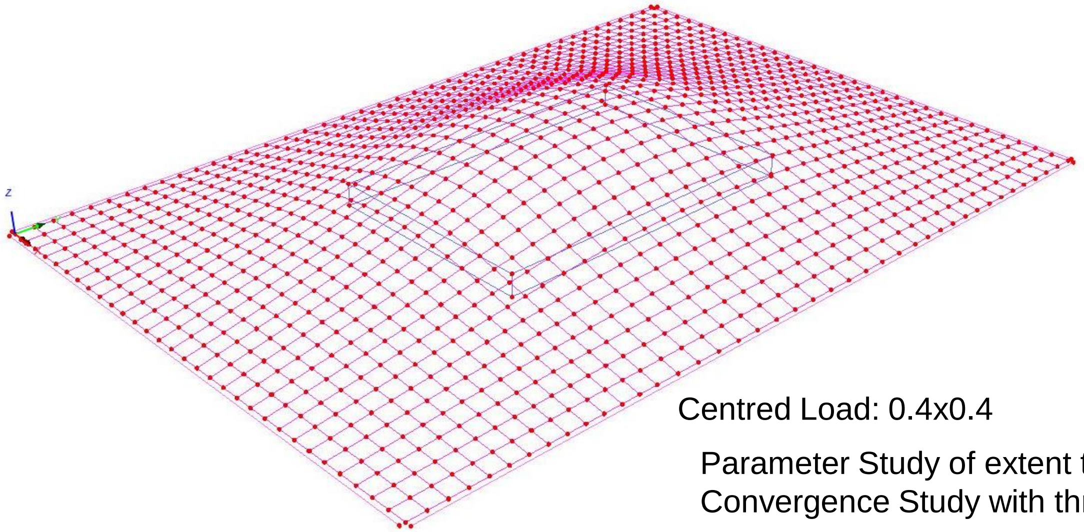


Centred Load: 0.6x0.6

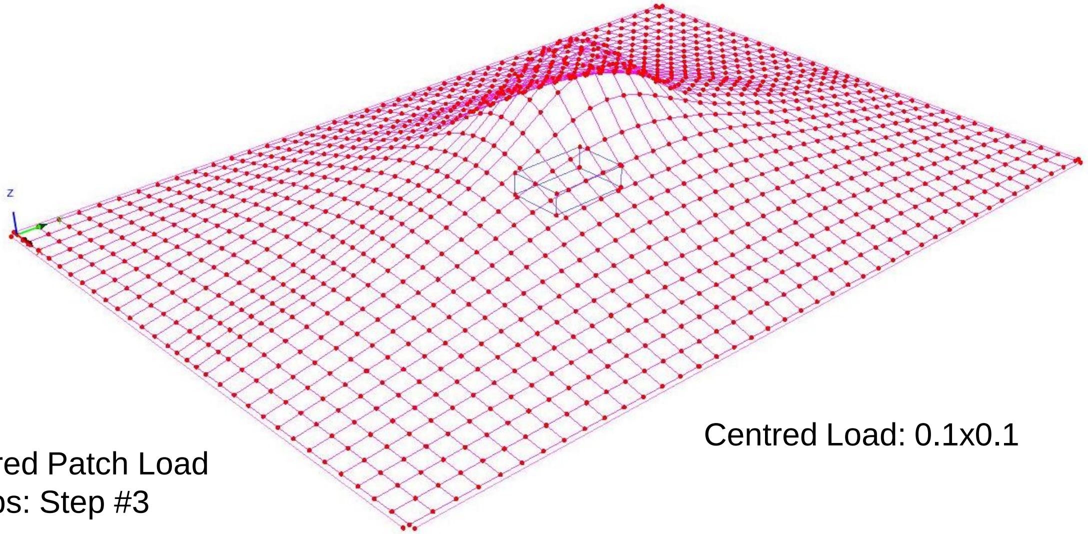


Centred Load: 0.2x0.2

Result Component: M_{xx} Bending Moment



Centred Load: 0.4x0.4



Centred Load: 0.1x0.1

Parameter Study of extent to Centred Patch Load
Convergence Study with three steps: Step #3

Time Out: Convergence Studies in Analytical Methods, eh …

CPT and any other PDE for rectangular plates have solutions expressed as W(x,y), a continuous function. The values calculated at any point across the plate will be the exact solution. Adding more points at intersection between curves will make no difference to the accuracy of the results. So, a convergence study for analytical methods is not meaning full.

Having said that, the accuracy of the analytical solutions depends on the number of terms used in the Fourier series.
The 10 different result components converge at different rates:

- W fastest
- Mxx, Mxy and Myy slower
- Qx, Qy and Vx, Vy the slowest.

The number of terms in the Fourier series should be set independently for each results component and the number of terms included should depend on the increment in value that each term contributes.

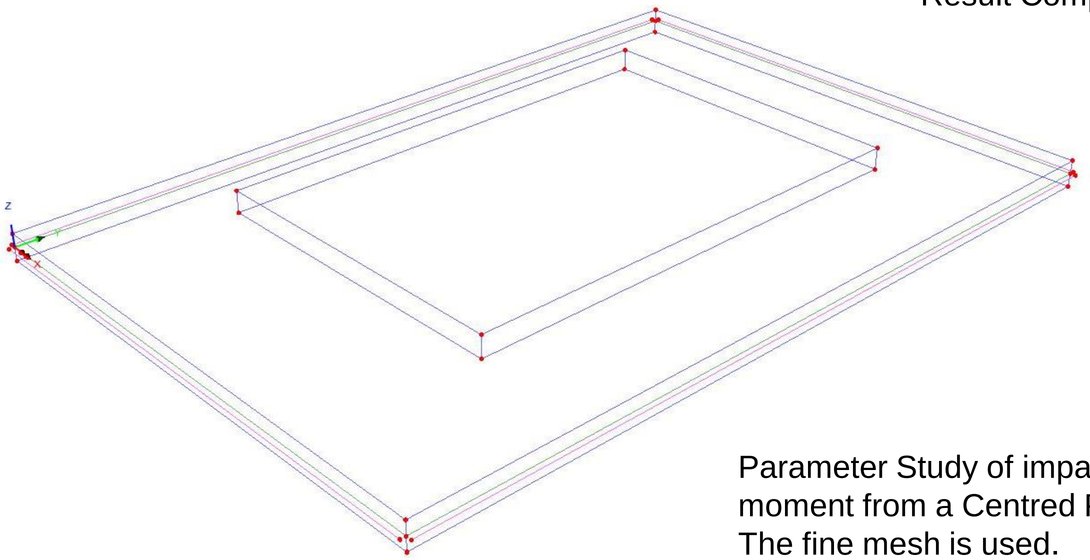
Of course, convergence studies make good sense for numerical methods like FEA where the number of DOFs (Degrees of Freedoms) or expressed differently, the size of the elements in areas of high gradients, determines the accuracy.

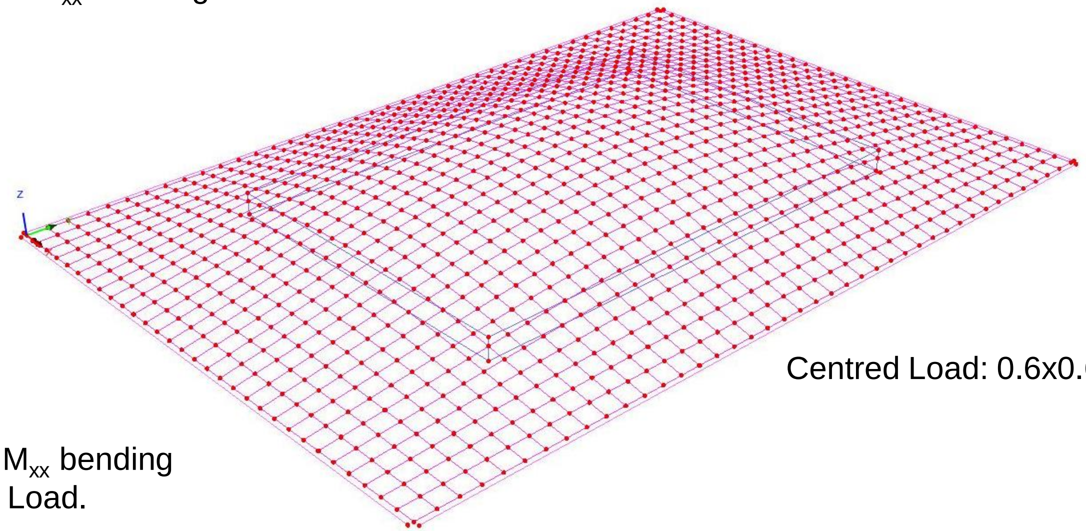
Comparing the two is necessary for the Verification activity in Verification and Validation working practices.

The examples in the previous slides should be complemented with FEA results and the Relative Error should be calculated.

A comprehensive range of ways of presenting the results: Example: Patch Load

Result Component: M_{xx} Bending Moment





Centred Load: 0.6x0.6

Parameter Study of impact on M_{xx} bending moment from a Centred Patch Load.
The fine mesh is used.

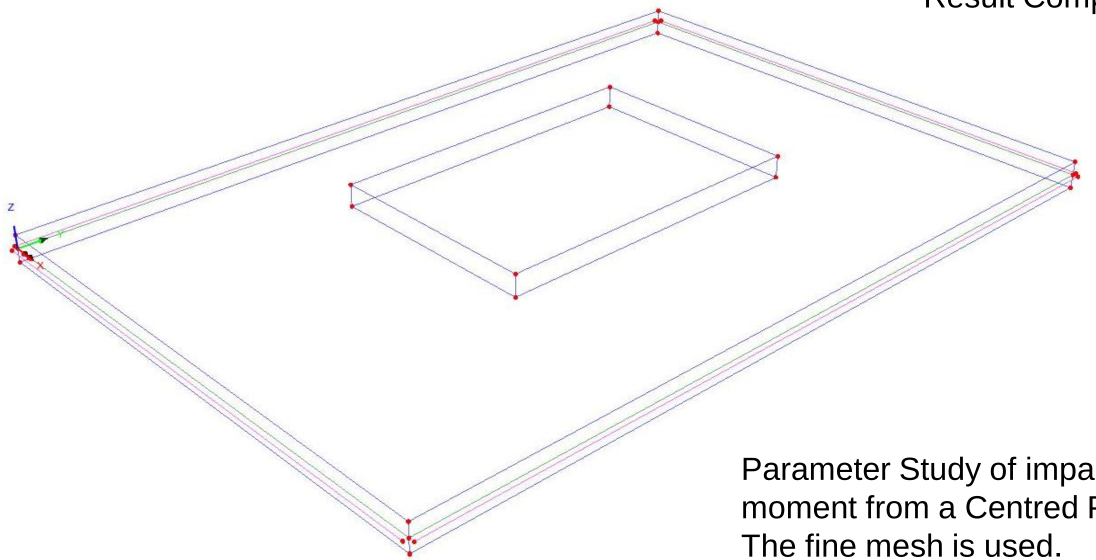
The presentation of results from analyses can be done in different ways.
In the previous pages, emphasis was made on the convergence of the analysis results based on alternative discretizations.

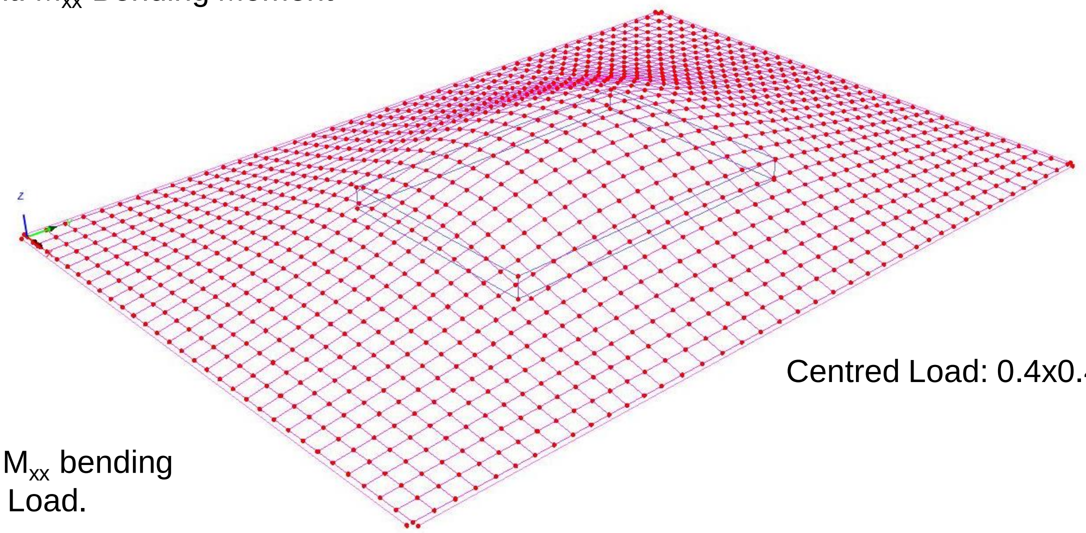
The same images can be put together with the emphasis on parameter changes, in this case the size of the patch load.

Here, a sequence of M_{xx} bending moment results are put together to show the start of the effect of moving from a full uniformly distributed load to a concentrated load, where the M_{xx} bending moment will increase towards infinity.

A comprehensive range of ways of presenting the results: Example: Patch Load

Result Component: M_{xx} Bending Moment





Centred Load: 0.4x0.4

Parameter Study of impact on M_{xx} bending moment from a Centred Patch Load.
The fine mesh is used.

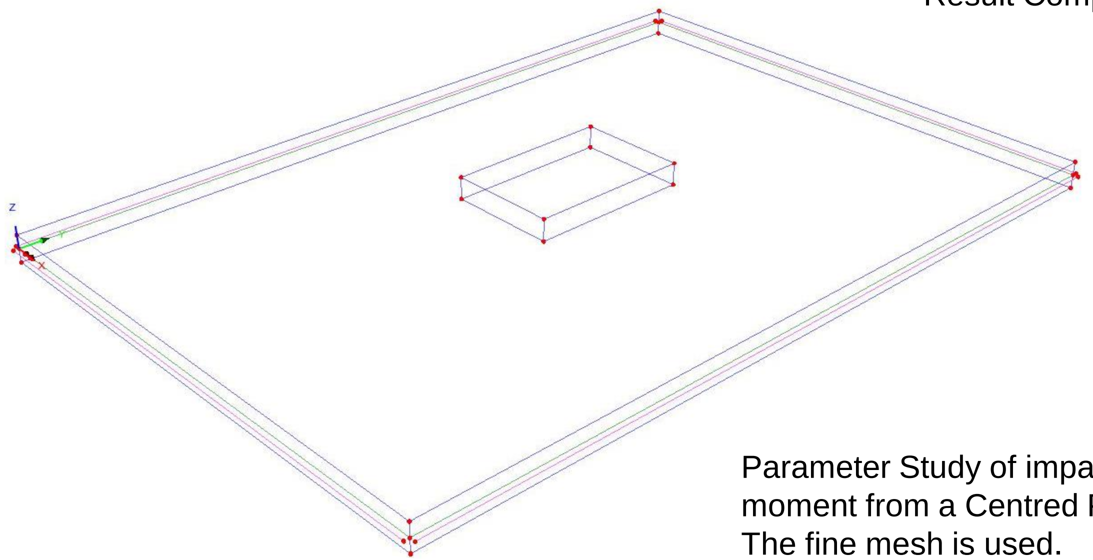
The presentation of results from analyses is best made as videos:

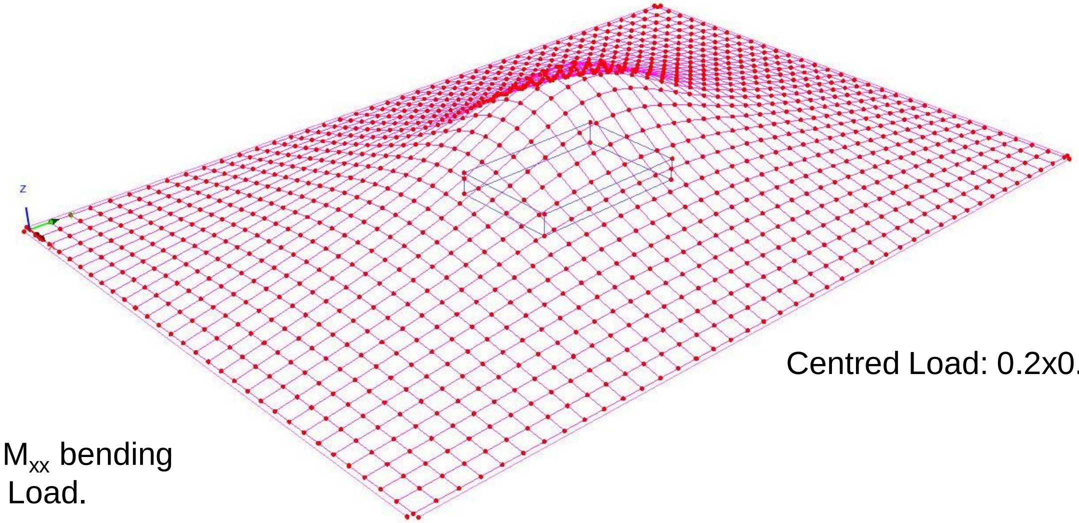
- a video will compress the time for a user assessing results;
- any number of result images can be put together to form a sequence, i.e.,
- each step in a sequence can have any number of changes from the previous (few) image(s), it is up to the user if they can recognise the pattern that the pictures reveal.
- Results from hundreds of analyses can be put together as a sequence

When the Old meets the New …

A comprehensive range of ways of presenting the results: Example: Patch Load

Result Component: M_{xx} Bending Moment





Centred Load: 0.2x0.2

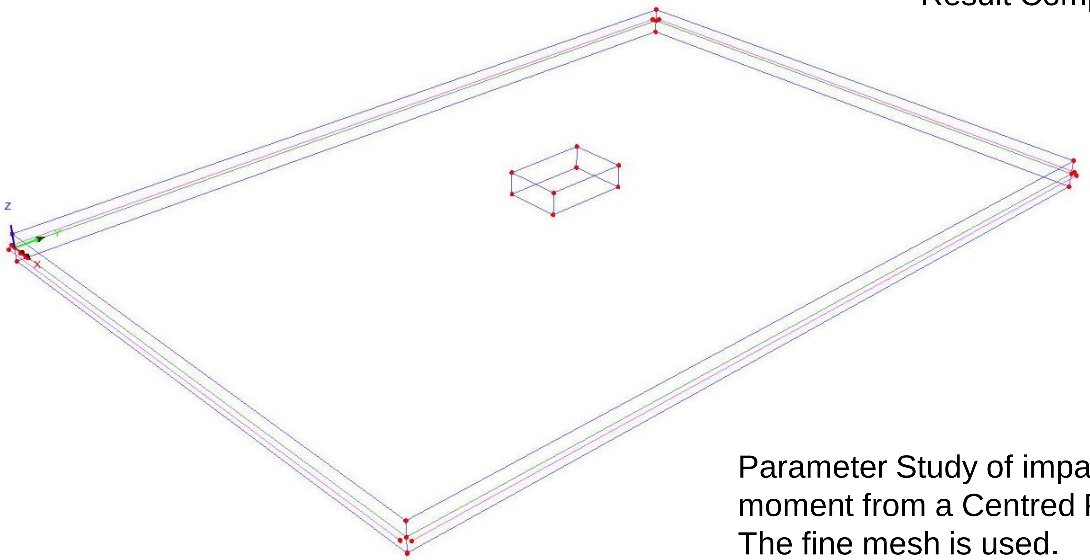
Parameter Study of impact on M_{xx} bending moment from a Centred Patch Load.
The fine mesh is used.

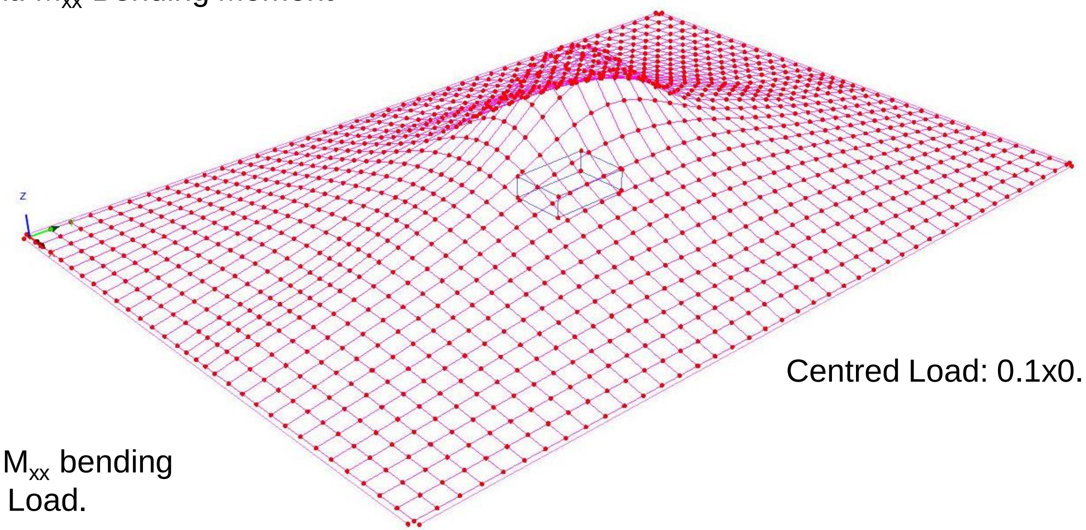
A video can be made in several ways:
- Putting together a number of pictures on successive slides in PowerPoint and play them as a Slide Show;
- Putting together a number of pictures as a sequence in video-making software like Windows Movie Maker for W10.
- The FEA pro-processor has a Create Animation function so a vbs script writer can put together and control the execution through scripts.

The author has made good progress in using all three of these methods to create a number of videos.
Time and space for this presentation prevent the use of videos this time round.

A comprehensive range of ways of presenting the results: Example: Patch Load

Result Component: M_{xx} Bending Moment





Centred Load: 0.1x0.1

Parameter Study of impact on M_{xx} bending moment from a Centred Patch Load.
The fine mesh is used.

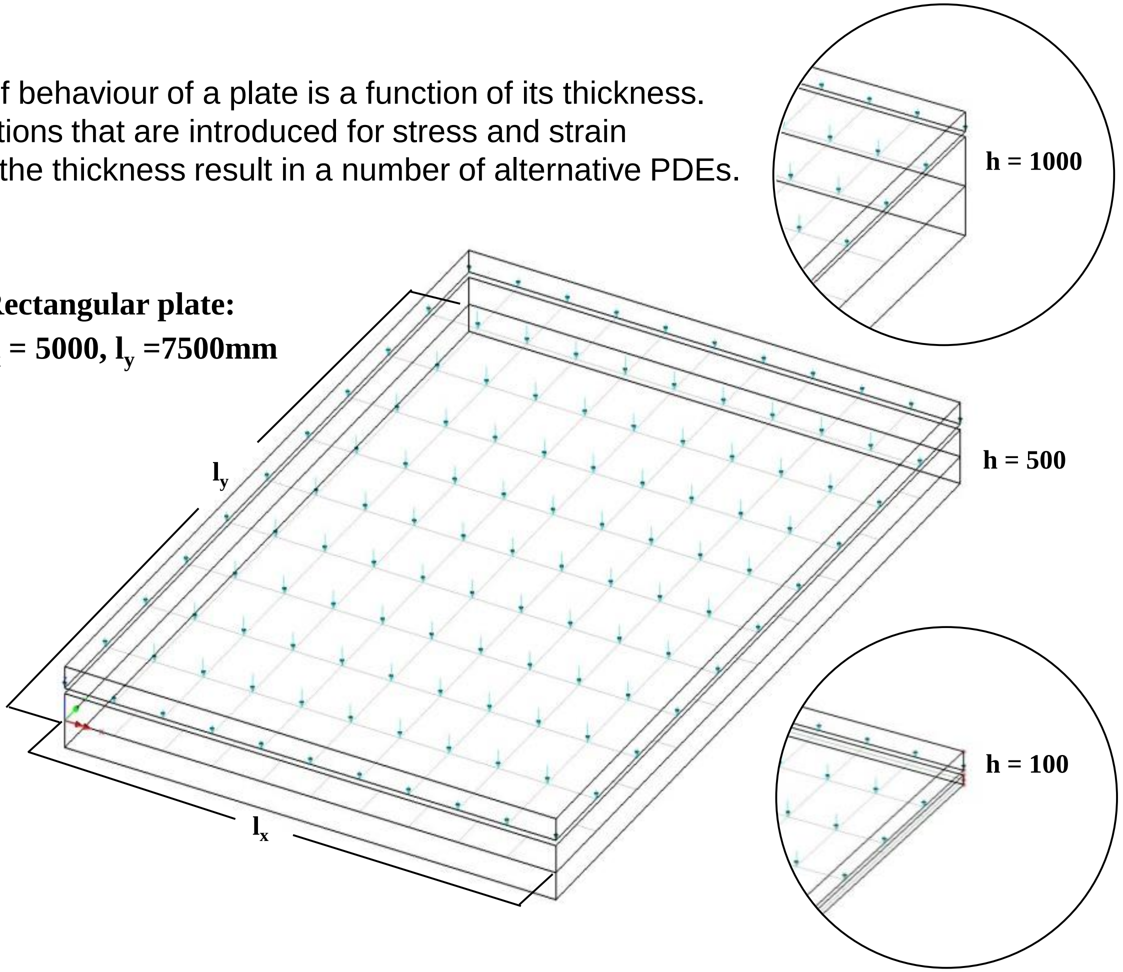
What happens next to the M_{xx} Bending Moment?

Videos are very effective communication tools for conveying an idea or a number of facts to any audience. Every analysis project should use video information as part of the project documentation to demonstrate that for example, the design criteria are met.

Handling of a range of mathematical models: Formulation

The definition of behaviour of a plate is a function of its thickness. The approximations that are introduced for stress and strain variations over the thickness result in a number of alternative PDEs.

Rectangular plate:
l_x = 5000, l_y =7500mm



3D Elasticity — **Thick plate**

$h/\min(l_x,l_y)$ = 1/5

Reissner-Mindlin — **Moderately Thick**

$h/\min(l_x,l_y)$ = 1/10

Kirchhoff-Love — **Thin Plate**
this is used for the CPT

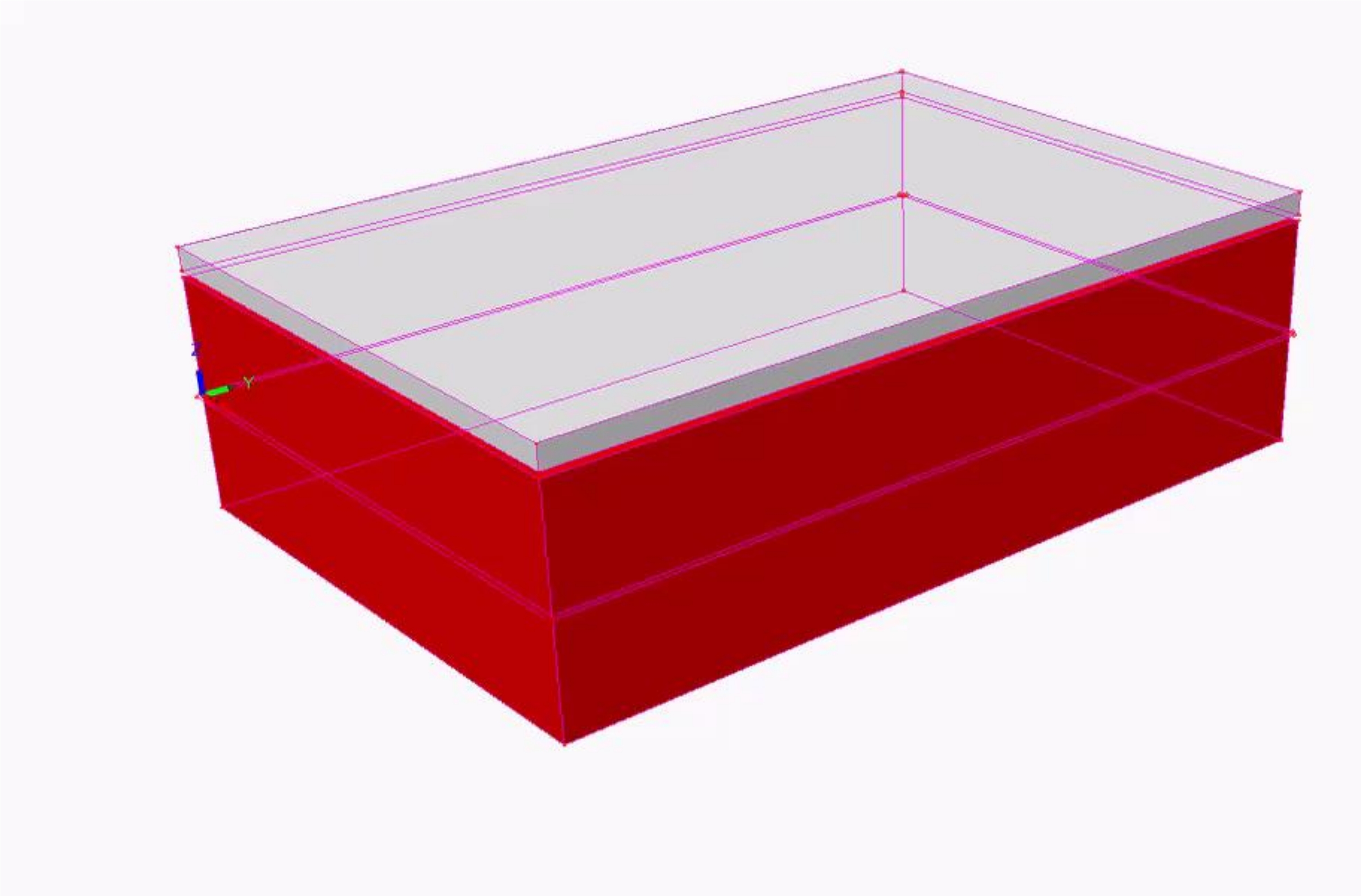
$h/\min(l_x,l_y)$ = 1/50

Föppl-Von Kármán — **Very Thin Plates**

Handling of a range of mathematical models: Formulation

The thickness range for CPT

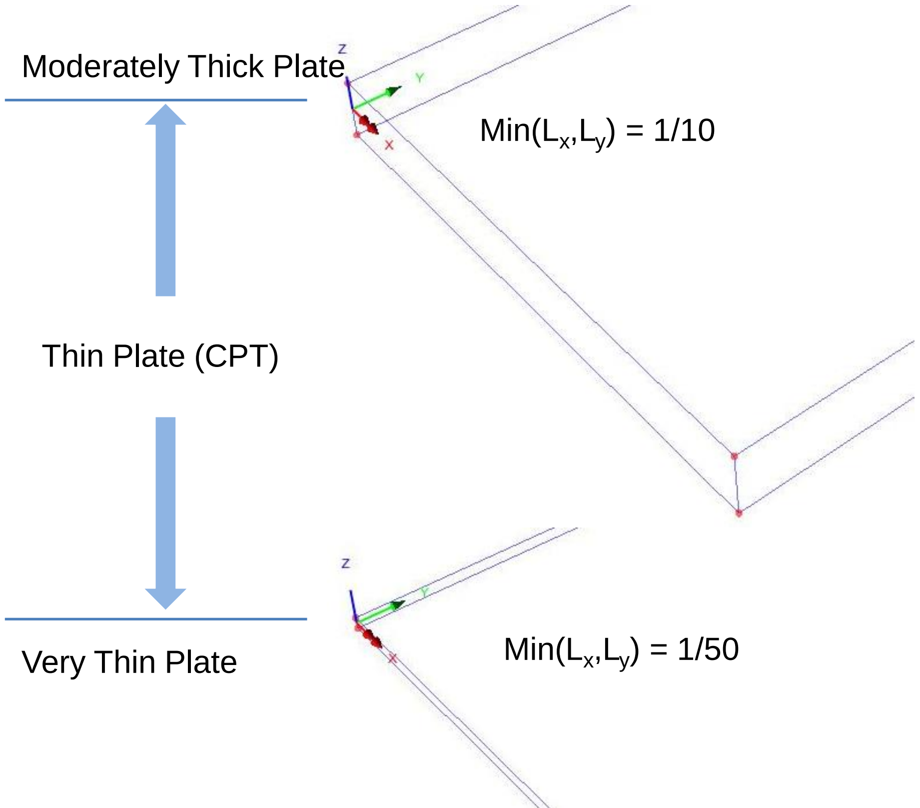
For a plate with nominal dimensions: 10.0 x 15.0 the thickness variations are explored



5.0 - 2.0	Very Thick Plate (i.e., 3D): The plate is best represented with the 3D elasticity formulation
2.0 - 1.0	Moderately Thick Plate: The plate is best represented with the Reissner-Mindlin formulation
1.0 - 0.2	Thin Plate: The plate is best represented with the Kirchhoff-Love formulation, i.e., CPT
0.2 - 0.1	Very Thin Plate: The plate is best represented with the Föppl-Von Kármán formulation

Handling of a range of mathematical models: Formulation

The thickness range for CPT



Plates made in concrete fall outside (above) this range.
Most plates made in steel fall within this range.
Many sheet metal applications fall outside (below) this range.

The CPT describing thin plates is by far the best covered in the literature.

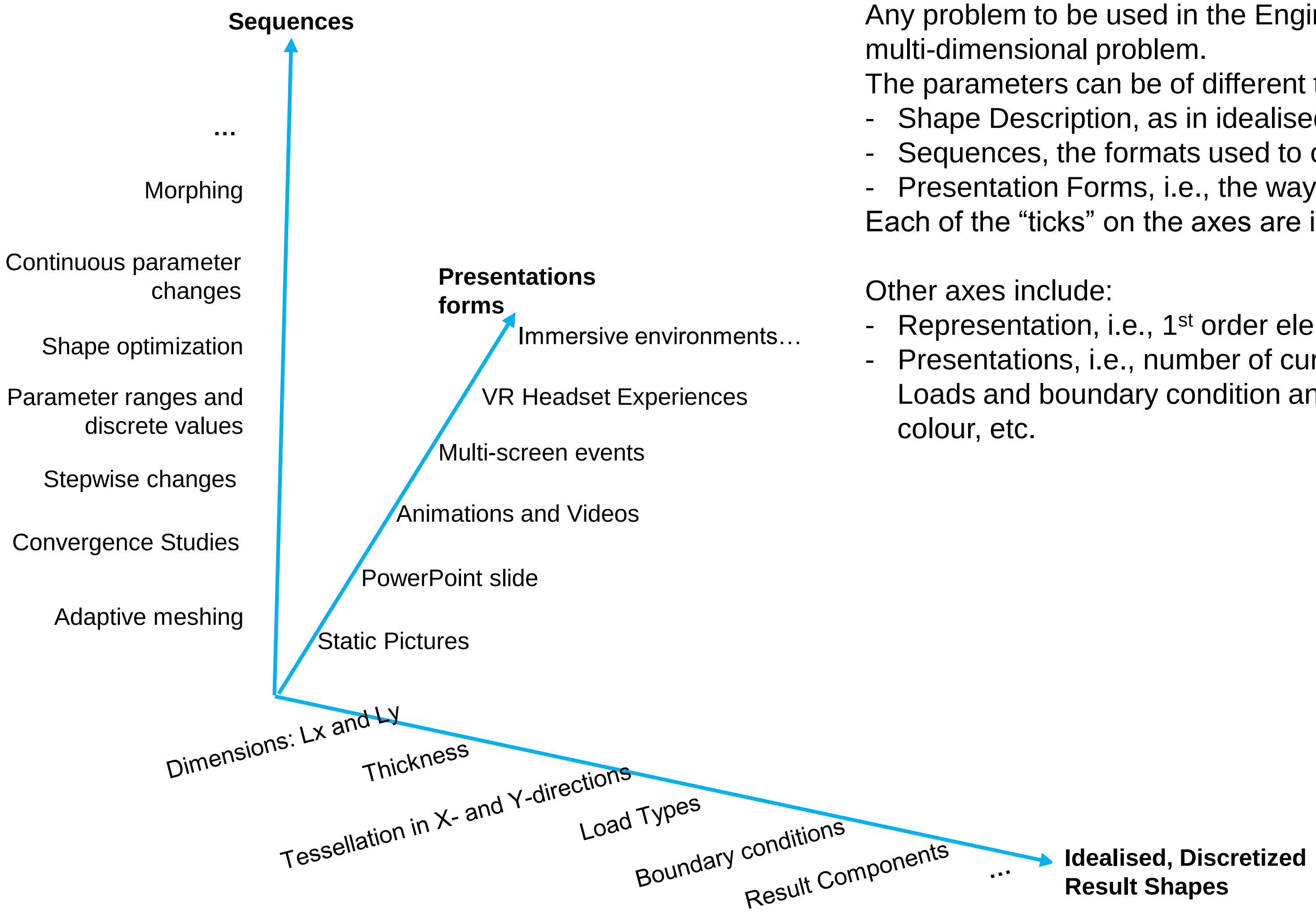
However, the CPT is based on assumptions that make it a particular case,
There are PDEs for generalisation in several directions:

- Thickness variation in x- and y, i.e., h = F(x,y) or D=F(x,y)
- Elastic foundation
- Anisotropic and orthotropic materials
- Membrane forces present
- Large deformations
- Buckling and dynamics

Rectangular Plate	**Moderately Thick**	**Thin**	**Very Thin**
Min(h/l_x, h/l_y)	1/5 to 1/10	1/10 to 1/50	< 1/50
Formulation	Transverse shear deformation included	Without Transverse shear deformation, popular for practical applications	Geometrically non-linear with membrane deformation
Plate Theory	Reissner-Mindlin	Kirchhoff-Love	Föppl-Von Kármán
Related Beam Theory	Timoshenko	Euler-Bernoulli	Theory of 2nd order

When the Old meets the New …

A multi-dimensional search space



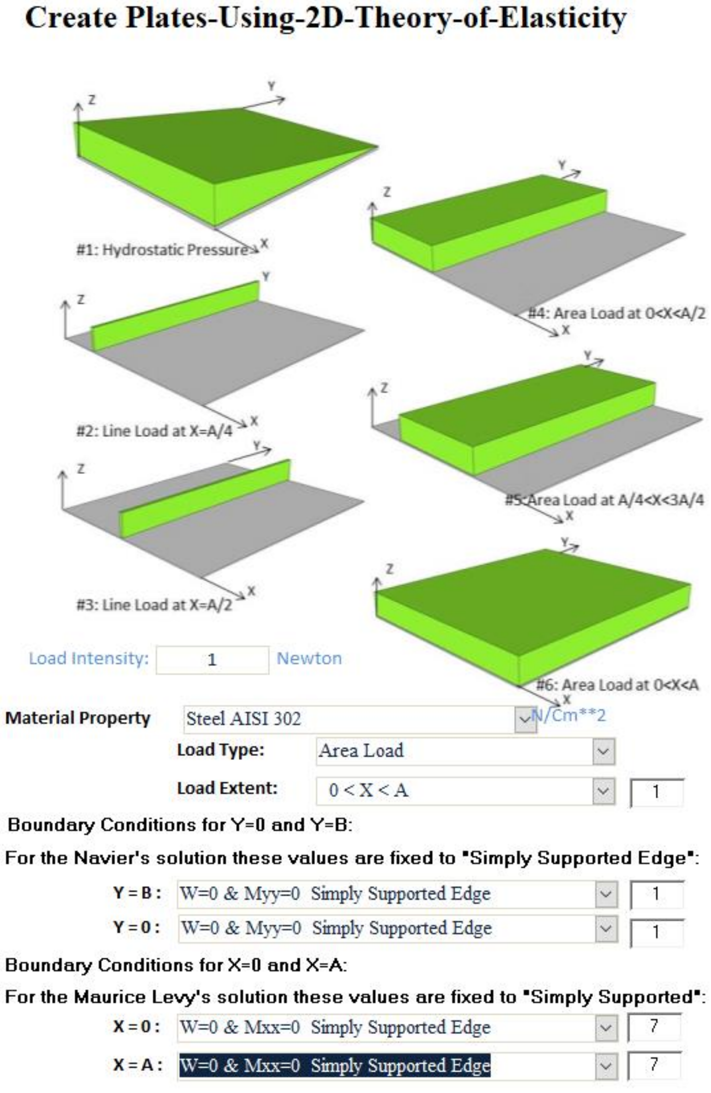
Any problem to be used in the Engineering Workbench must be defined as a multi-dimensional problem.
The parameters can be of different types:
- Shape Description, as in idealised, discretized and results shape
- Sequences, the formats used to create many analyses;
- Presentation Forms, i.e., the way sequences are presented;

Each of the "ticks" on the axes are in themselves a parameter space.

Other axes include:
- Representation, i.e., 1st order elements, 2nd order elements and beyond
- Presentations, i.e., number of curves in the pictures, surfaces included, Loads and boundary condition and other ways the shape presented in colour, etc.

When the Old meets the New …

A multi-dimensional search space



Specify discrete values for variables:

Specify ranges for variables (from a separate application for AHM):

The Engineering Workbench has already 20-odd variables to specify and control. There are many more to come. A user can specify the individual parameter as a discrete value or a range of values.

The different plate PDEs require to specify different sets of parameters, for example compare

- Isotropic rectangular plates
- Orthotropic rectangular plates

Engineering analysts should be supported by the software so they can be an advisor to an automatic or semi-automatic process and make decisions based on good judgement and experience. The software should do what computers are good at: making solutions.

A multi-dimensional search space: Examples of use

The analytical results and the FEA results can be presented as animations and/or videos where each step or picture contains a delta-change to the content of the previous picture, stringing together results from a (large) number of analyses. By going through a range of values for a number of parameters, a large (or small) search space can be created and results brought together in an animation, a video presentation or in a real time display.

A number of animations belong at this point in the presentation. However, space and time constraints have made it inappropriate to include them at this point.

The following stories are under development and/or still on the idea stage:

- “How accurate is the EXACT solution?” comparison of analytical results where the number of Fourier terms determines the accuracy, and the FEA results where the size of the elements determines accuracy.

- “From Simply Supported to Clamped Edges, all around the plate”, sequences where the simply supported edge is gradually made into a clamped edge by adding a bending moment in the range (0-1) at the edge. This is the superposition principle used in practice, extensively promoted by T&WK, by turning a simply supported plate to a fully clamped plate by going through all stages and perturbations.

- “From uniformly distributed load to point loads”, how do the various result components respond?”

- “Navier’s, Levy’s and FEA solution methods, how do they compare when solving the same problem?”

- “Isotropic and orthotropic plates, how do they behave?”

- “How far in goes the influence of a boundary condition?” For any plate, the effect of any boundary condition is a local effect. How does the different boundary conditions influence the bending moments and stresses away from the edge?

What would you like to know? Suggestions are welcome.

Time-out: The story so far:

Summary of the findings so far, based on their “best endeavour”:

The authors of most of the books present results by other’s and develop the theories further to a point where they are ready for implementation. However, at the time they developed these theories and published the books:

- They didn’t have the means to implement the theory to solve practical problems themselves;
- They did their best with what they got: logarithmic tables, slide ruler (slipstick) (also based on logarithms), hand calculations, etc. and subsequently advise the readers to “take it from here …”

- They expected the interested reader to take the next step:
  To actually implement the theory to get answers they are looking for in their ongoing projects.

A pertinent remark:
The study of the theory of rectangular plates has highlighted the importance of Formulation for assessing result from calculations.
I have come to the realisation that Verification and Validation should more appropriately be called Formulation, Verification and Validation, F, V & V, to truly reflect the significance of Formulation in good working practices.

Control question:
When did you last consult the theory manual for you favourite FEA system to check the element type(s) you are using, to assess what effects they can cater for?

Time-out: The story so far:

Summary of the findings so far, based on their “best endeavour”:

In practice, no one has the time and energy to implement the theory in a way that results in a practical tool.

- Engineers are practical people, and need to solve their design problems within time and cost constraints.

- University courses in 2D Elasticity Theory focus on the theory (again), leaving the students to be tested on the CPT in their exams. They are not taught understanding of the behaviour of plates and trained in solving plate problems using the theory.

Substantial effort is required to complete the development of the theory: all load types, all boundary conditions, all plate thicknesses (that is: alternative formulations), all solution methods. In particular, to take the theoretical work into the realm of “practical problems”.

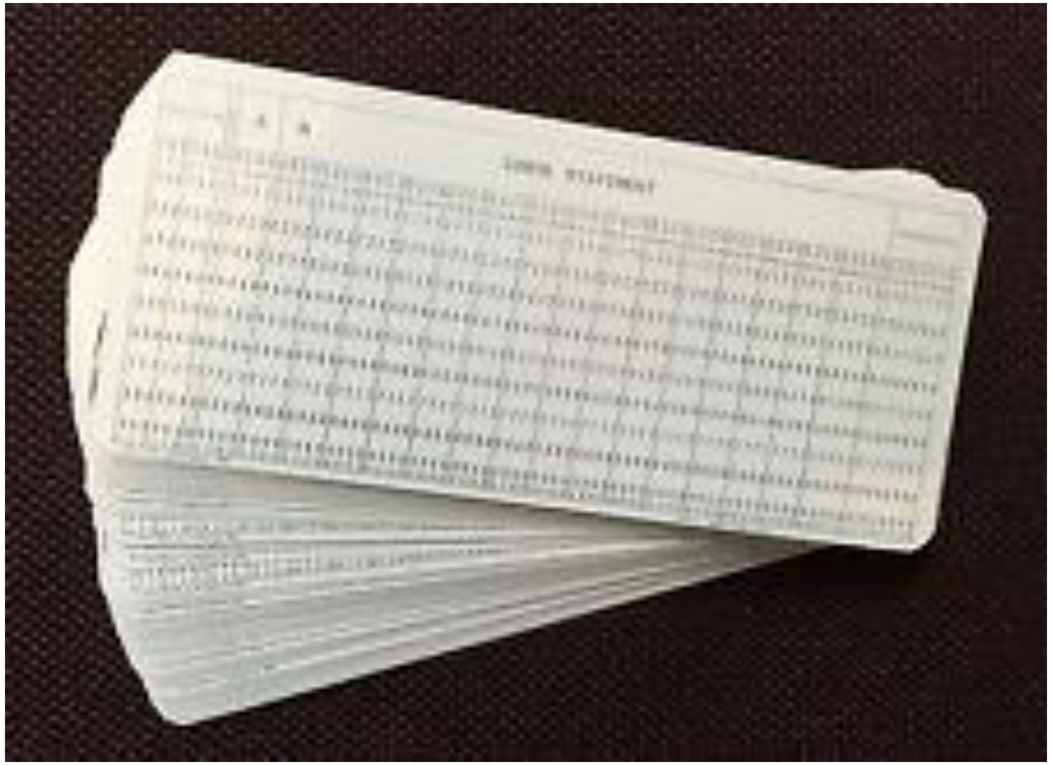
In addition, an implementation must use modern office automation tools and methods, including RDBMS systems like MS Access and SQL Server.
Once a software solution is working, the result should be made available to others to use, actually solving practical problems in the design of plates (and shells). It will be a major support tool for Formulations, Verification & Validation.

In stead, these magnificent theories are left on the shelf to gather dust and to be remembered with fondness, while engineers solve plate and shell problems with four-noded tetrahedra, lots and lots of them … (well, it’s not that bad everywhere …)

We can do better than that …

When the Old meets the New …

A modern user interface: From Batch Jobs to Games



Card deck for software development and analysis input from 1960s onwards, (“Why did we throw them all away? … “)
In principle they are still in use as input and output files for whatever:

- IGES, STEP, DXF, etc.
- Analysis input and output files
- Graphics files for plotters and graphics terminals
- A long list …

Xbox Elite Wireless Controller for Windows 10 and Xbox One, and the next and the next and …

The future, no questions asked …

When the Old meets the New …

A modern user interface: From Batch Jobs to Games

The "Old in computing"
1960s-1970s

The "New in computing"
2000s-2010s

The "Quite Recent",
1981 - …

IBM PCs got their breakthrough thanks to SuperCalc

In the new world of computer games, users (gamers) are used to respond to immediate understanding of the information passing in front of their eyes (and ears).

A Card punch machine
and
IBM 1403 line printer, the classic line printer of the mainframe era.

Yes, engineering analysis results need to be represented in rows and columns for quantitative assessment and presentation of the results, (read: spread sheets).
However, that is only one way of presenting the information.

https://www.bing.com/videos/search?q=final+score+live&&view=detail&mid=AF40FA4E74A8C0757A67AF40FA4E74A8C0757A67&&FORM=VRDGAR

When the Old meets the New …

A modern user interface: From Batch Jobs to Games

Wherever we look we are bombarded with graphical information. We are used to moving images on TV, - both programs and commercials -, in movies, in Internet browsers and apps, in computer games, etc.
We are good at interpreting and absorbing the information and finding meaning that often is covered up and not obvious to the casual viewer.

Why is the presentation of engineering analysis results not making use of our skills in interpreting graphical information?

In between these two ends of the spectrum from the previous slide, there are ways where information is conveyed in a multi-media environment:

- Live coverage of sport events of TV such as “Final Score” on BBC1 on Saturday afternoons, where pundits comment live on what is going on in a number of football matches (soccer) while banners run at the bottom of the screen and results are displayed for a number of divisions continuously to the left. This is real parallel processing.

- A news reader or whether forecaster presents information from a number of sources by touching a large screen. New pages are opening up and closing down on touch, information can be static displays, moving images and live streaming of events.

Behind these presentations are a frantic activity by many contributors, ready when called up on.

A modern user interface: From Batch Jobs to Games

A typical trader in financial markets

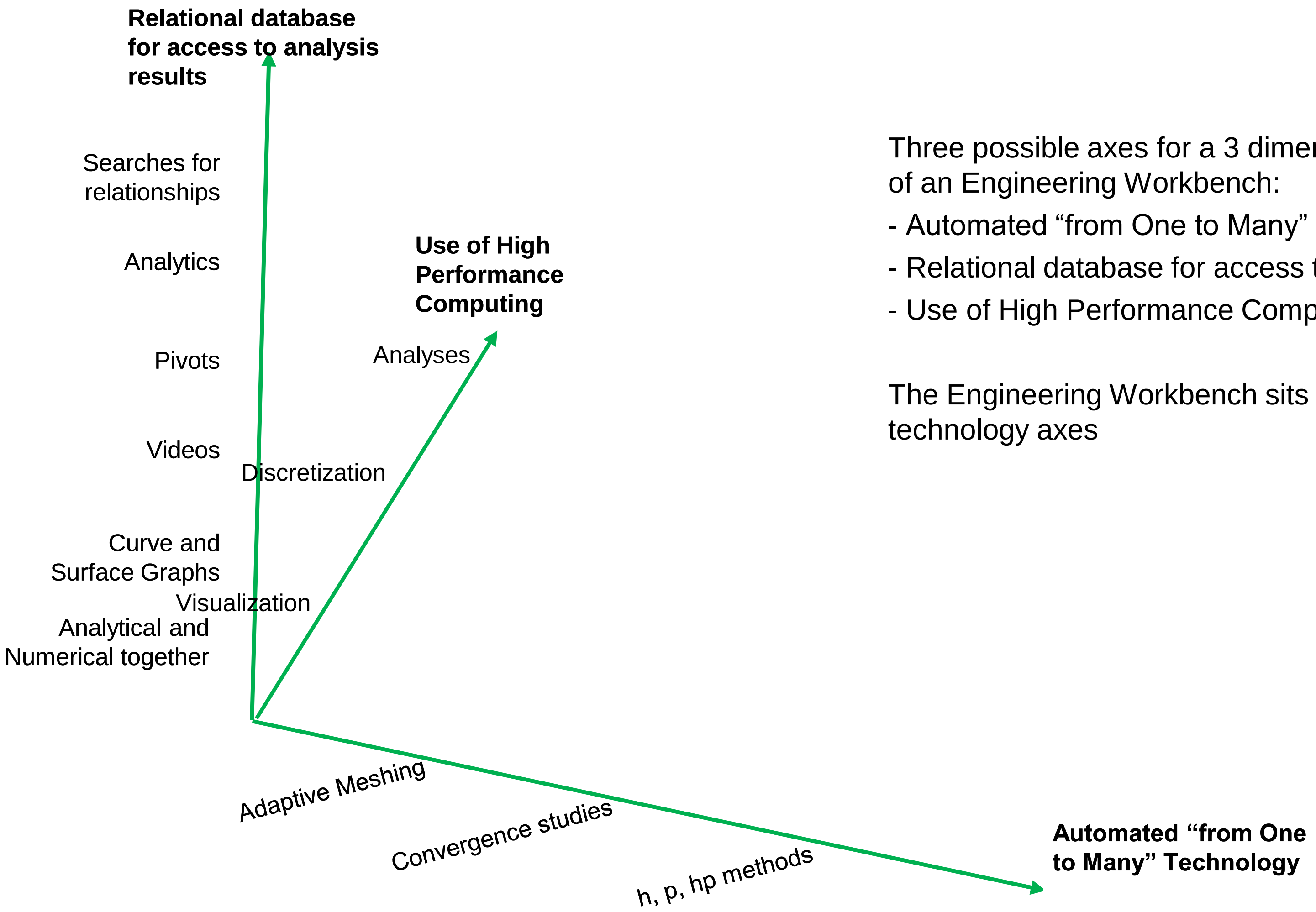
It has struck me that I want the analysis processes and the analysis results presented to me in the same way as a financial market trader works, running 20+ analysis sequences simultaneously and get the “scores” from each presented as they complete and new analyses are started. I want to compare results across analyses in real time.

Why can’t we do this for engineering analysis?

The opportunity is missed because:
- software developers are unable or unwilling to advance their software functionality and
- end-users are unable to change established working practices.

Or is it only because they cannot imagine what is possible with state of the art technology?

A blue sky opportunity



Three possible axes for a 3 dimensional space defining the functionality of an Engineering Workbench:

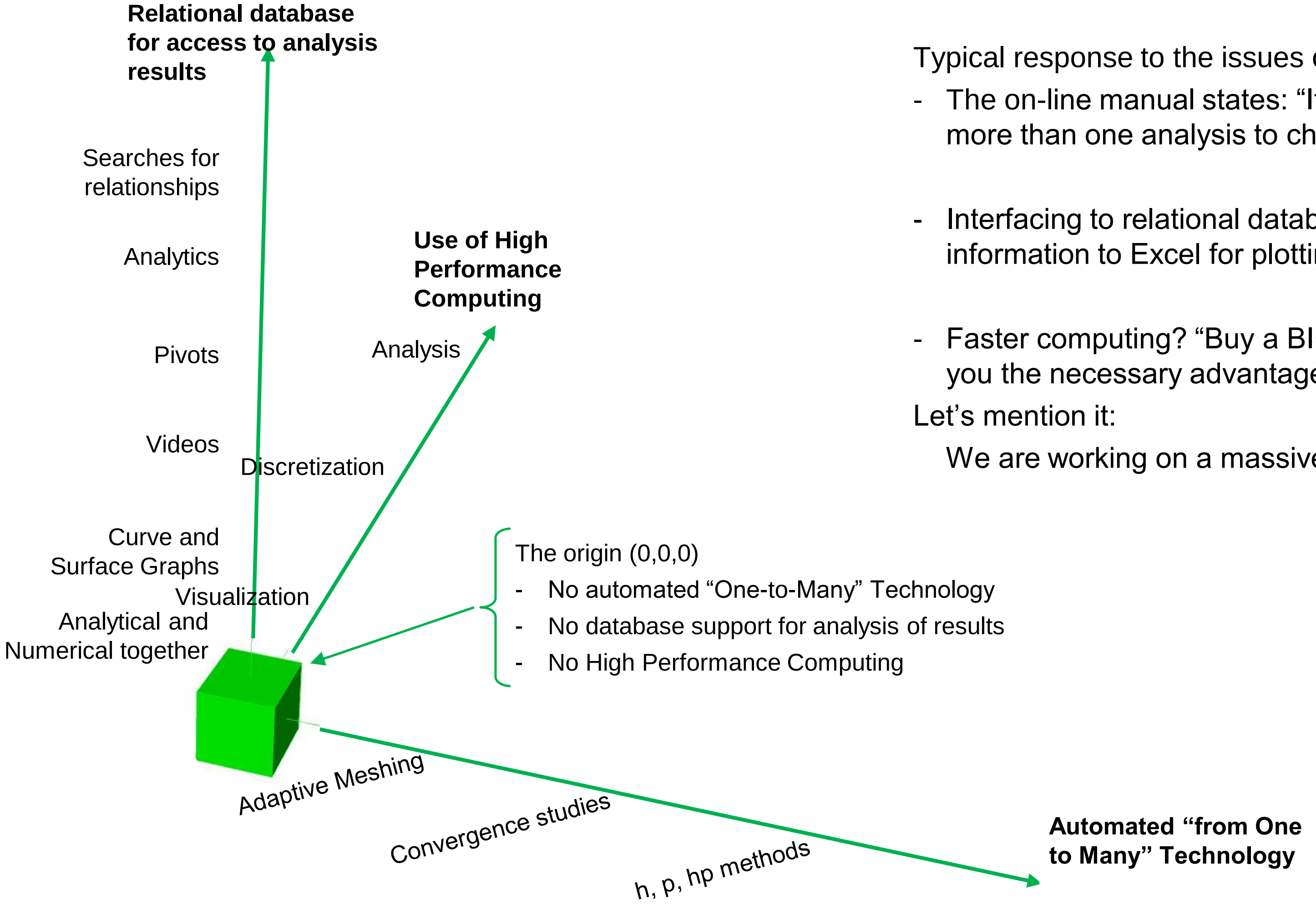
- Automated "from One to Many" Technology
- Relational database for access to analysis results
- Use of High Performance Computing

The Engineering Workbench sits in the space spanned by these technology axes

A possible 3 dimensional space spanning technologies for the Engineering Workbench

When the Old meets the New …

A blue sky opportunity



A possible 3 dimensional space spanning technologies for the Engineering Workbench

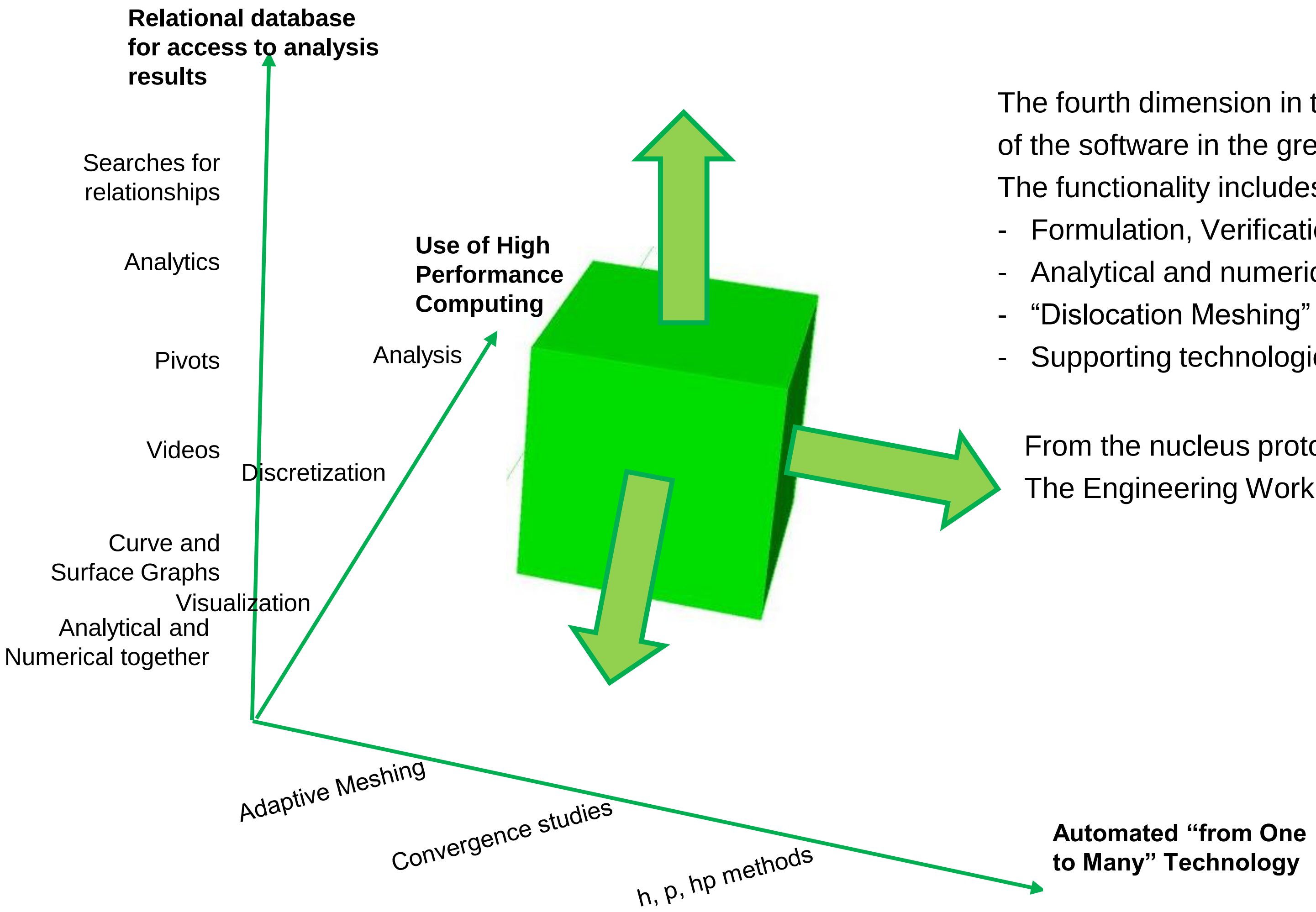
Typical response to the issues on each of the axes:

- The on-line manual states: "It is recommended that users run more than one analysis to check the accuracy of their results" …
- Interfacing to relational databases? "You can send some result information to Excel for plotting of a graph"
- Faster computing? "Buy a BIGGER computer" (Moore's law gives you the necessary advantage) and …

Let's mention it:

We are working on a massively distributed equation solver …

A blue sky opportunity



A possible 3 dimensional space spanning technologies for the Engineering Workbench

The fourth dimension in this 3D axis system is the functionality of the software in the green box: The Engineering Workbench
The functionality includes:

- Formulation, Verification, & Validation support
- Analytical and numerical analyses integrated
- "Dislocation Meshing" for Automatic Hexahedral Meshing
- Supporting technologies for activities along all three axes

From the nucleus prototype demonstrated here, the functionality of The Engineering Workbench can expand in all three directions.

A blue sky opportunity

The engineering content of this presentation may seem simple …
linear static analysis of rectangular plates using the CPT formulation, Navier's solution method and thin plate elements for FEA simulations.

However, the goal is to explore the opportunities that open up when exploiting well-established computer technology to engineering analysis using rectangular plates.

The applications can be expanded in a number of directions, spanning an n-Dimensional search space:

- Alternative shapes for a rectangular plate, including thickness variations in x-, y- and z-directions
- Alternative loads and boundary conditions
- Alternative material representations, isotropic, orthotropic, anisotropic, etc.
- Alternative analysis types like buckling, dynamics, variational representation, sensitivity analyses, etc.
- Alternative Formulations, methods for Verification and links to experimental results for Validation
- Any other full 3D problem and shapes using reduction in dimension: plates, membranes, all sorts of shells, beams, bars, cables, etc.

And

- Anything non-linear, geometry, material, deformation, loads, boundary conditions, you name it …

Oops, did I leave out something? …

Question: What analytical solutions exist for 3D Formulations?

When the Old meets the New …

A blue sky opportunity

The expansion of existing and the development of new analytical solutions will facilitate better F, V & V working practices.

The introduction of automatic convergence studies will support the verification process in F, V & V.

The introduction of multi-analysis search spaces will enable users to explore alternatives and create their own experience.

The use of modern computers and software development tools will:

- enable practicing engineers to understand the behaviour of < any structure > so they can design better solutions;
- encourage the practical side in teaching of mathematical modelling at universities;
- motivate youngsters (read: gamers) looking for a meaningful and interesting career to become mathematical modellers.
  ref. (STEM – Science, Technology, Engineering, Mathematics)
  https://en.wikipedia.org/wiki/Science,_technology,_engineering,_and_mathematics

A quote to be heard everywhere soon:
“I enjoy my time at work because I can play computer games called mathematical modelling all day.“

When the Old meets the New …

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- motivate youngsters (read: gamers) looking for a meaningful and interesting career to become mathematical modellers.
  ref. (STEM – Science, Technology, Engineering, Mathematics)
  https://en.wikipedia.org/wiki/Science,_technology,_engineering,_and_mathematics

A quote to be heard everywhere soon:
“I enjoy my time at work because I can play computer games called mathematical modelling all day.“

When the Old meets the New …

A blue sky opportunity

The expansion of existing and the development of new analytical solutions will facilitate better F, V & V working practices.

The introduction of automatic convergence studies will support the verification process in F, V & V.

The introduction of multi-analysis search spaces will enable users to explore alternatives and create their own experience.

The use of modern computers and software development tools will:

- enable practicing engineers to understand the behaviour of < any structure > so they can design better solutions;

- encourage the practical side in teaching of mathematical modelling at universities;

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Conclusions

We owe it to the originators of the underlying mathematics for Classical Solid Mechanics to demonstrate what they could have done themselves if they only had the same computer technology as we have.

Like them, we are stuck in our own time and technology, prevented from moving forward by our own minds.
Our children and grandchildren will have even larger and more advanced opportunities than we have.

There is now a clear separation in the Current and the Future, expressed as a divide in computer use defined by age:

- Before the arrival of computer games, i.e., using line-oriented input devices
- After the arrival of computer games, i.e., using games controllers as the input device

There is no doubt about it, the engineering analysis industry as a whole:

- University teachers
- Software developers
- Engineering managers
- Practicing engineers

have to adopt modern computer technology in all its forms:

- to adopt Formulation, Verification and Validation working practices
- to advance mathematical modelling as a game
- to define and execute analyses in multi-dimensional search spaces, “Many small analyses”
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As an example, the first Exa FLOPS (Floating Point Operations Per Second) machines are predicted available in 2021, computers that can do >1,000,000,000,000,000,000 FLOPS.
What do you plan to use them for?

Plan A: "More of the same"
That is, to continue batch oriented working practices of "ONE big analysis" where LARGER analyses can be done FASTER and at a LOWER cost.

Plan B: "A new way of working"
That is, fully adopt the opportunities that we are offered by today's computer technology to develop software and working practices of "MANY small analyses", to embrace Formulation, Verification & Validation so you can combine analytical and numerical analyses and harvest the results across analyses, formulations and solution methods in real time.

Scaling and tuning of the HPC technology and software implementations will in time create "MANY big analyses".

HPC will open up for parallel processing at an unimaginable scale, compressing week-long analysis sequences to become interactive.

What do you plan to do today so you are ready when Exa FLOPS machines arrive?

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I want Pica FLOPS, or better, Exa FLOPS, computer power hanging off the tip of my games controller, with the software functionality to compute, capture and present the results coming out of multi-dimensional simulation sequences in real time.

Most of the computer technology needed is here already,
When will software for engineering analysis catch up?

The early version of the Engineering Workbench is just a start, and the presentation here clearly shows that this is work in progress, there is a lot more to do in all sorts of directions.

However, it is the start on something that can grow very large: in effort, in involvement, technical coverage and in usefulness.

I know where I am going, do you want to come?

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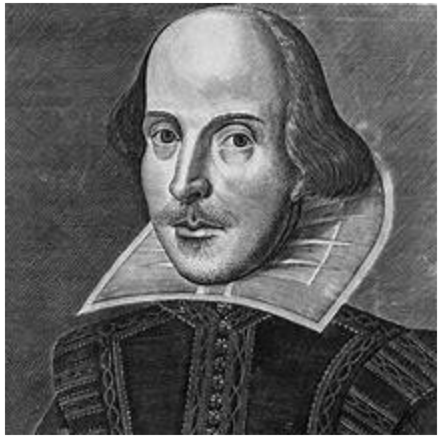
Brutus delivers this speech to Cassius in Act-4, Scene 3 of *Julius Caesar by William Shakespeare (1564-1616)*.

Brutus:
“There is a tide in the affairs of men.
Which, taken at the flood, leads on to fortune;
Omitted, all the voyage of their life
Is bound in shallows and in miseries.
On such a full sea are we now afloat …

And we must take the current when it serves,
Or lose our ventures.”

(Julius Caesar, Act-IV, Scene-III, Lines 218-224)



William Shakespeare (1564-1616)