DISLOCATION MESHING –

A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

Jon Aas, CT Innovations Ltd

Monty Roberts, The man who talks to horses:

- As a teenager he studied the behaviour of wild Mustangs in the Nevada Desert
- He learned the behaviour patterns that horses used all the time
- Adopting the same body language, he made even wild Mustangs respond in “Join-Up” with him
- and willingly take a saddled rider within 20 mins of meeting.
- again and again, no flukes …

Monty Roberts called his language **Equus®**

He is now running a very successful family business Flag Is Up Farms in Solvang, CA, teaching other horse trainers his methods to an international audience.

Check it out: https://www.montyroberts.com/

Monty Roberts, with his dear friend Shy Boy

How can his “story” be repeated?

DISLOCATION MESHING –
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Discovery of patterns in behaviour

Mathemization of the patterns

Algorithms based on these patterns

The author has studied the behavior of quadrilateral and hexahedral meshes at length and

- found the patterns in the behavior of quadrilateral and hexahedral meshes
- formalized these into a mathematical language
- implemented these patterns into algorithms

The result is Dislocation Meshing, a general language for tessellation and meshing of surface and solid models

Well-formed mesh flows

well-formed elements with minimum distortions

Recommended "best choice" mesh

Dislocation Meshing allows quadrilateral and hexahedral meshes behave the way that comes naturally to them:

- To offer the user well formed mesh flows with well formed elements;
- All with minimum distortion to meshes and individual elements;
- To recommend "best choice" quadrilateral and hexahedral meshes.

The behavior of quadrilateral and hexahedral meshes is a **"Law of Nature"**, you cannot avoid it however hard you try!

The analogy to Monty Roberts was too good to miss!

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CAD

A Mathematical Continuum

FEA

Alas, I got distracted, because a more general representation of shapes rouse up from the note pad in front of me,

- A general theory for representation and manipulation of shapes that unifies a number of disjointed technologies (software applications). These includes:

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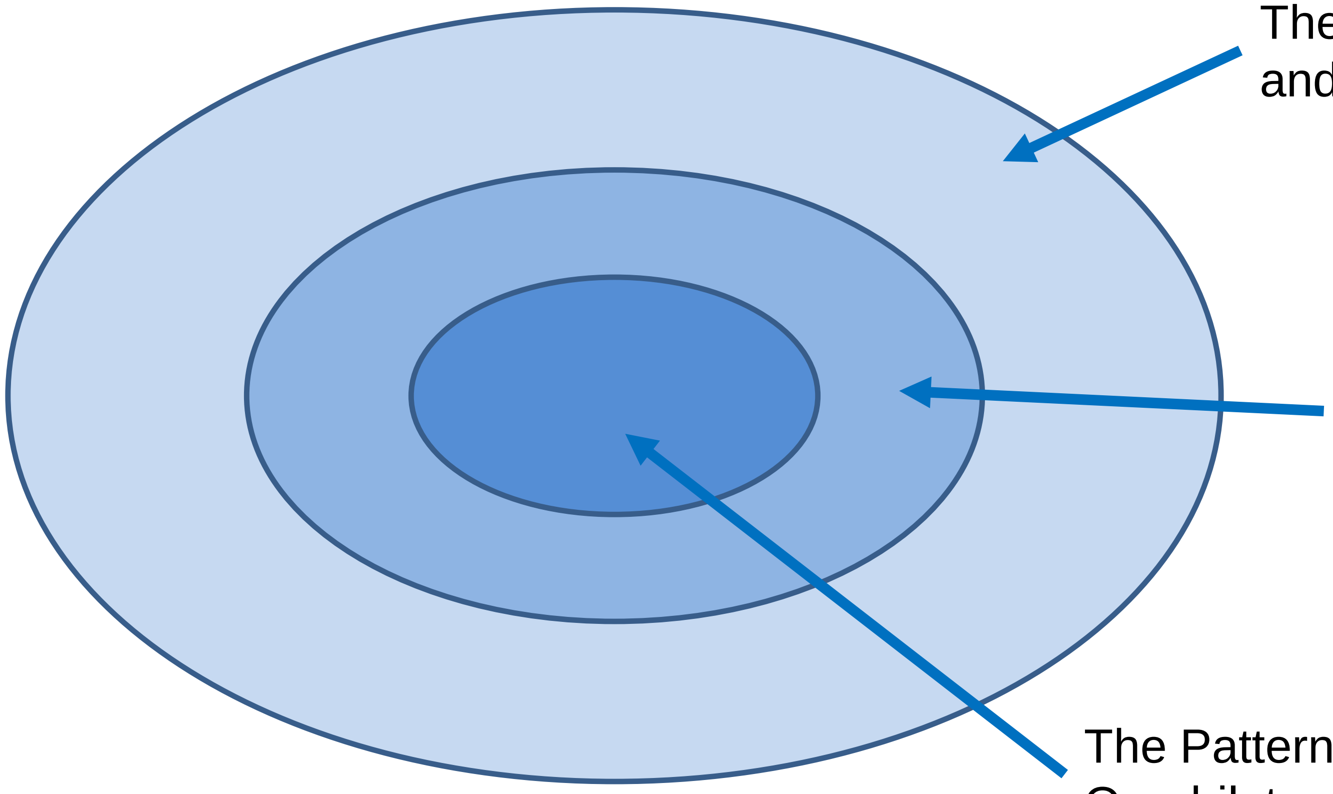
Dislocation Meshing is just one of many application of the more general theory.

The way the research results relate



Dislocation Meshing: The presentation

- Why call the behaviour patterns Dislocation Meshing?
- What to use it for: Examples
- The General Theory of Shape Representation
- A Mathematical Continuum
- What about Formulation, Verification & Validation?
- Discussion
- Conclusion

A balanced presentation of the material requires more time than the allotted 20 mins.
So a compromise: A comprehensive presentation has been developed, and I will hop, skip and jump through it to focus on examples.
Anyone interested in learning more?

- Read the paper
- Peruse the whole presentation
- Ask questions
- Check out the Facebook Page: www.facebook.com/CT.Innovations.NWC19 where more examples are uploaded
- Eventually, e-mail us at: jon.aas@ctinnovations.co.uk

At the NAFEMS World Congress ’97 in Stuttgart, 9-11 April 1997

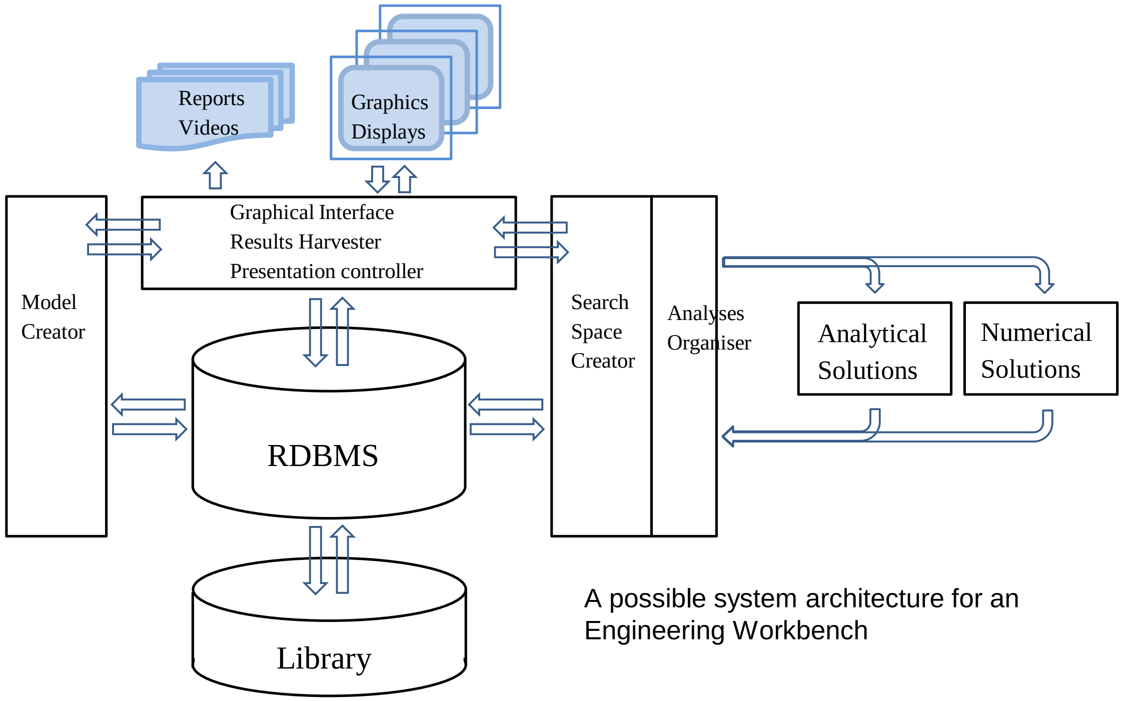
Dr Bruce E. MacNeal and Dr Richard H. MacNeal gave a presentation called **“Future Issues – a Code Developer’s Perspective”** where they stated:

“The “holy grail” of automatic hexahedral meshing still eludes us. Automatic meshing technology will have a long growth phase.”

And so it has proven …

By April 1997, I had already worked on the understanding of the patterns that control the behaviour of quadrilateral and hexahedral meshes for 12 years.

My Career Goals



A possible system architecture for an Engineering Workbench

As a young engineer I realised that I had to solve the problem of Automatic Hexahedral Meshing to get to where I wanted to go:
To develop an Engineering Workbench where FEA analyses could be run effectively using the best technology available for interactive studies of "change":

- alternative formulations for the PDEs we wanted to solve;
- alternative solution methods to get reliable results, using analytical as well as numerical methods;
- alternative discretizations where the result quality was no longer sensitive to the underlying meshes;
- the results could be correlated with other data, verifying their accuracy and relevance to the real world problem we sought to simulate;

- To "play" with the analysis models;
- To use alternative boundary conditions, load types and cases, material properties which could be changed systematically to study the effect of change;
- anything else that would come up during my structural engineering career.

Of course, this leads straight to Formulation, Verification and Validation working practices.

Why call the behaviour patterns Dislocation Meshing?

Early discoveries: The Patterns

- Quadrilateral and Hexahedral meshes have two components: create the structure of the mesh, position the nodes in a 3D space
    - The Structure of a mesh is topology,
    - Positioning of the nodes is geometry;

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In the process, I have “broken all the rules” for how meshing algorithms work, there is no connection or incremental improvements to any other approaches for automatic hexahedral meshing.

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Dislocation Meshing stands on its own, this is replacement technology, a paradigm shift in approach to the problem.

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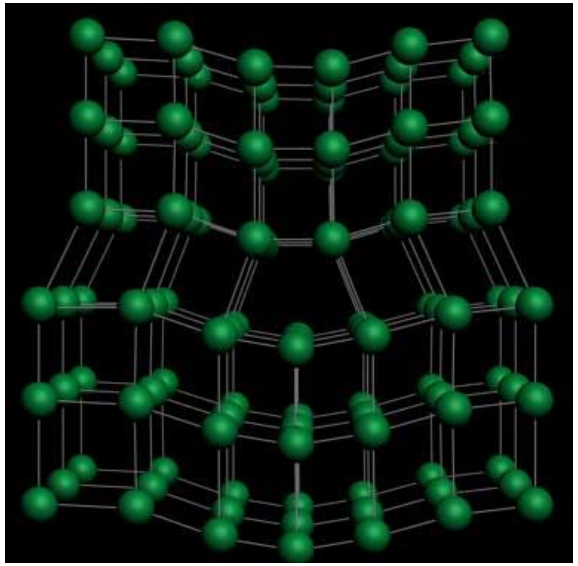
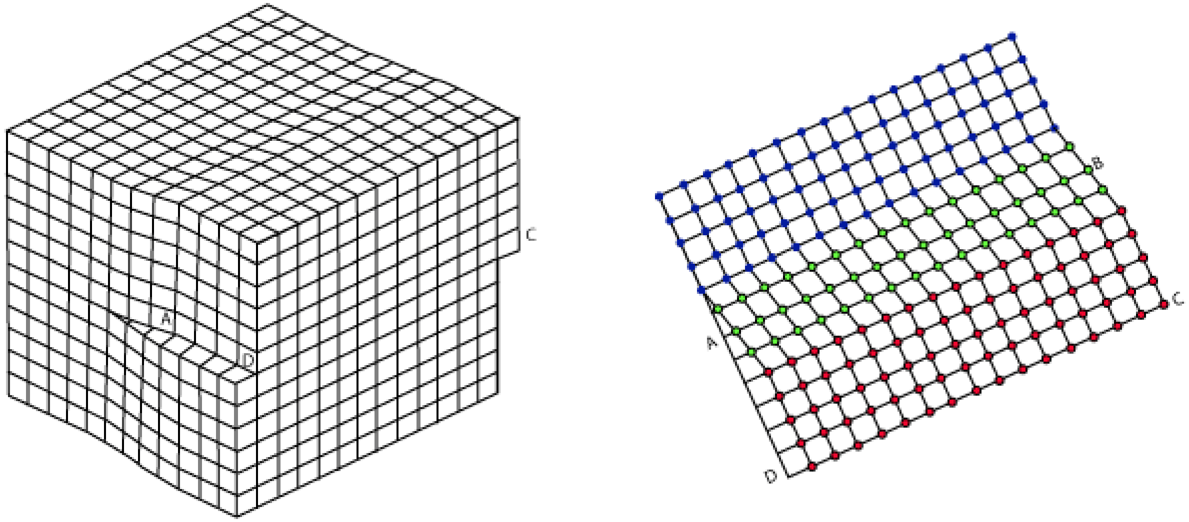


Schematic visualisation of an edge dislocation



Schematic visualisation of a screw dislocation

The name Dislocation Meshing has been pinched from metallurgy where irregularities appear in regular crystal lattices.
In metallurgy, a **dislocation** is “a defect in a metal lattice which occurs when a few atoms in a layer are missing”.

“Two types of dislocations exist: **edge and screw dislocations**. Dislocations found in real materials are typically mixed, meaning that they have characteristics of both.”

To find further details you can search the Internet using the search term “dislocations in metal lattices”.

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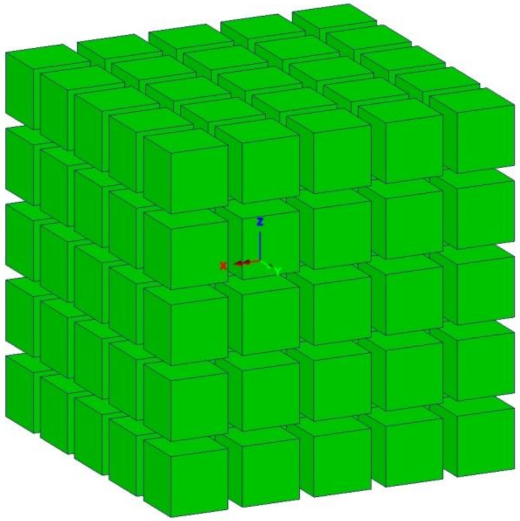
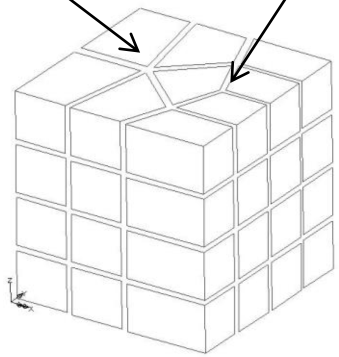
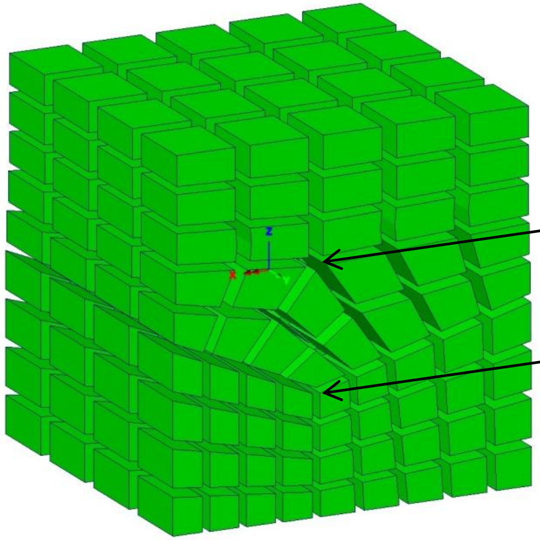


A regular hexahedral mesh in a cube using Hex-in-Hex regular meshing





A hexahedral mesh in a cube with a dislocation pair (a three-way and a five-way irregular edge) making a transition in one direction.
This example uses one of many Hex-in-Hex transition meshing patterns

Dislocations in finite element meshes are also “defects” from the regular hexahedral mesh in a cube.
Hexahedral meshes may have two types of irregularities:

- vertices and edges with one element less that the regular four, i.e., three-way irregularity and
- vertices and edges with one element more than the regular four, i.e., five-way irregularity.

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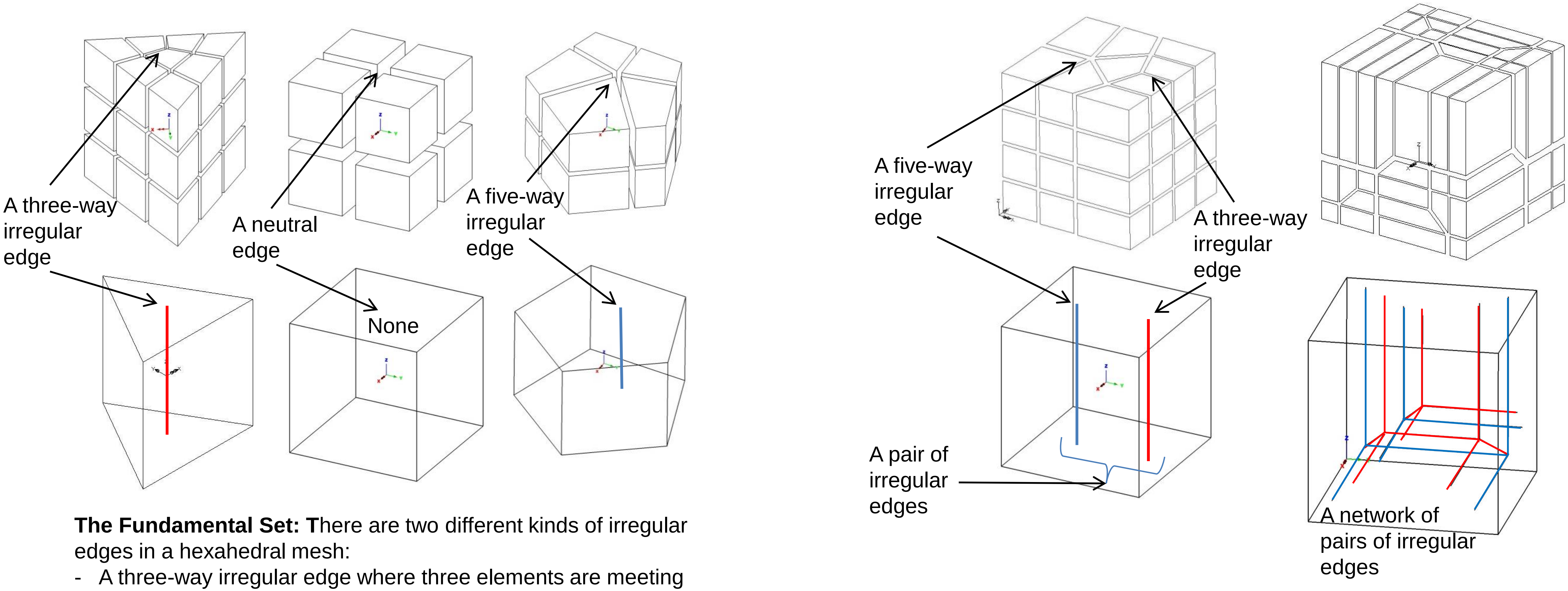

The Fundamental Set: There are two different kinds of irregular edges in a hexahedral mesh:

- A three-way irregular edge where three elements are meeting
- A five-way irregular edge where five elements are meeting

These three prisms form the basis for Full Separation of the irregular edges in a hexahedral mesh.

The number of irregular edges in a volume is defined by its boundary, i.e., the number of irregular vertices in the faces in its exterior.

The Auxiliary Set: Pairs of three-way and five-way irregular edges form structures in the interior of a volume and can interact with other pairs in a variety of ways. Their appearance is independent of the topology of the shape, and can be used any number of times in a hexahedral mesh.

Why call the behaviour patterns Dislocation Meshing?

The patterns that control the behaviour of the irregular edge networks:

Irregular edge networks in a hexahedral mesh	Irregular Edge Networks in the Fundamental Set	- Three-way irregular edges - Five-way irregular edges	These are ALWAYS present
	Irregular Edge Networks in the Auxiliary Set	- Pairs of three-way and five-way irregular edges interacting	These may be added

The Fundamental Set consist of individual irregular edges connecting two irregular vertices in faces in the external boundary of a shape; The irregular edges starts at an external irregular vertex, goes into the volume and pass other irregular edges or interact with other edges and come out again at a vertex on a different external face. An irregular edge connects two irregular vertices of the same kind.
When irregular edges interact with others, they form networks that connect external vertices in well-defined structures, those that are formed in any of the 3D Kernels. Any other networks are excluded to avoid poor mesh flow and poor element shape quality.
There are cases where the irregular edge networks form an internal loop, see the Truncated Trapezohedrons.
The Auxiliary Set is independent of the topology of the shape and can form irregular edge networks with other pairs in the Auxiliary Set in a variety of ways. There is in fact no upper limit.
The Dislocation Meshing algorithms introduce the Auxiliary Set as a transitional Hex-in-Hex meshing method after the Fundamental Set has formed the Coarse Hexahedral Cluster. In this way, the two sets of irregular edge networks are kept apart restricting the structures that can be created, and avoiding excessively complicated structures with poor mesh flow and poor element shape quality.

The behaviour of the irregular edge networks is a “**Law of Nature”** that controls the structure of quadrilateral and hexahedral meshes.

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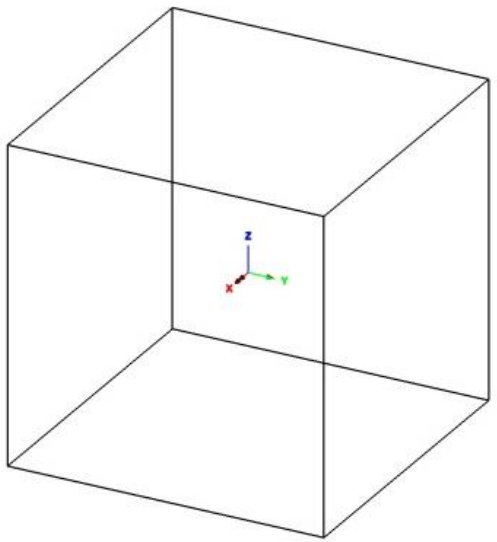
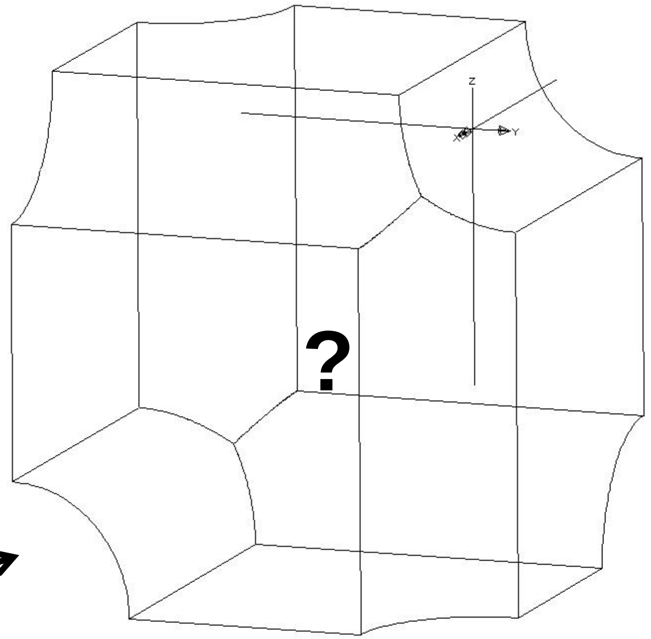


A shape bounded by only quadrilaterals hence there is no need for irregular vertices to create a hexahedral mesh. no irregular vertices or edges needed, i.e., well-formed elements throughout

But what about this?



A cube is the only shape where no irregular vertices and edges appear using a regular Hex-in-Hex meshing algorithm

An example of a geometric shape where irregular vertices and edges are necessary to create a hexahedral mesh

The patterns that have been found to govern the behaviour of quadrilateral meshes in 2D extends directly into 3D, controlling the behaviour of hexahedral meshes.

For a 3D hexahedral mesh to exist, the irregular vertices on the exterior must go into the volume, forming irregular edges.
These irregular edges must pass through the volume, they may or may not interact with other irregular edges and come out again at another face.

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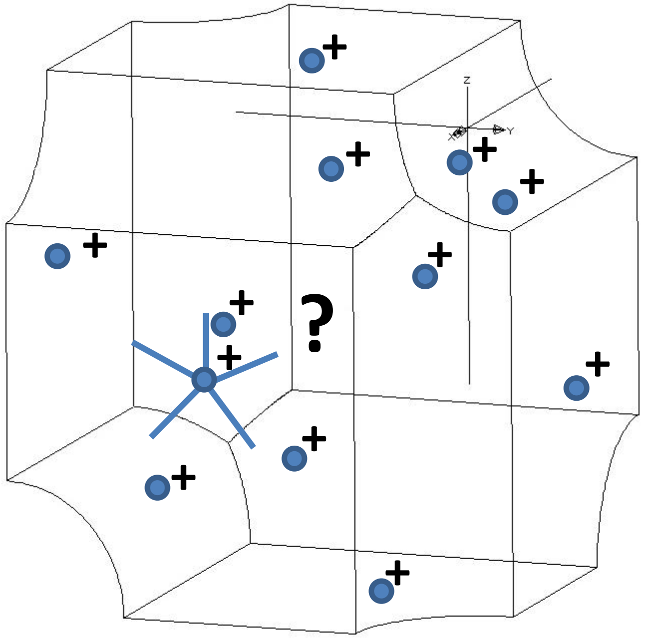
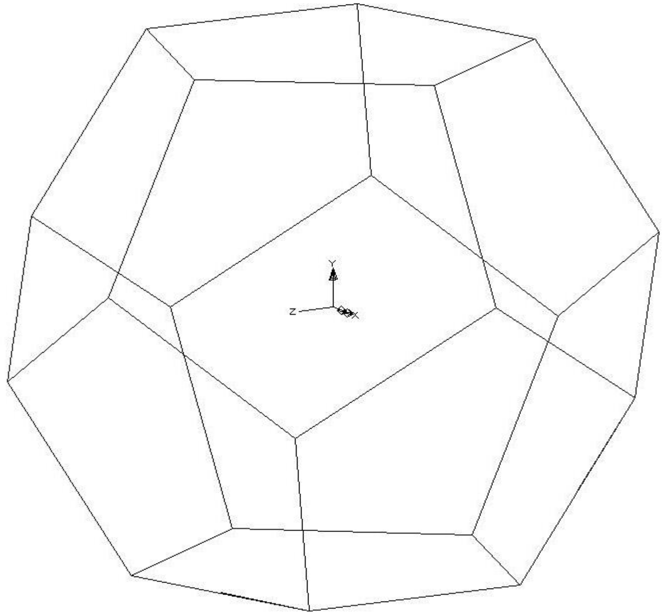


What do you see?

The topology of this geometric shape is visualised as a polyhedron with 12 pentagonal faces: the Dodecahedron



This shape is bounded by 12 pentagons, each having a minimum number of irregular vertices: one five-way irregular vertex each, 12 in all.

$$D = V_2 - 4$$

This is a Dodecahedron with a topology consisting of:

- 20 three-valent vertices;
- 30 two-valent edges;
- 12 pentagonal faces;

Euler's Formula for Polyhedra applies:

$$V – E + F = 2$$

A Dodecahedron is bounded by twelve pentagons. The actual shape of the topology is immaterial, but a regular Dodecahedron is now used to visualise the relationships.

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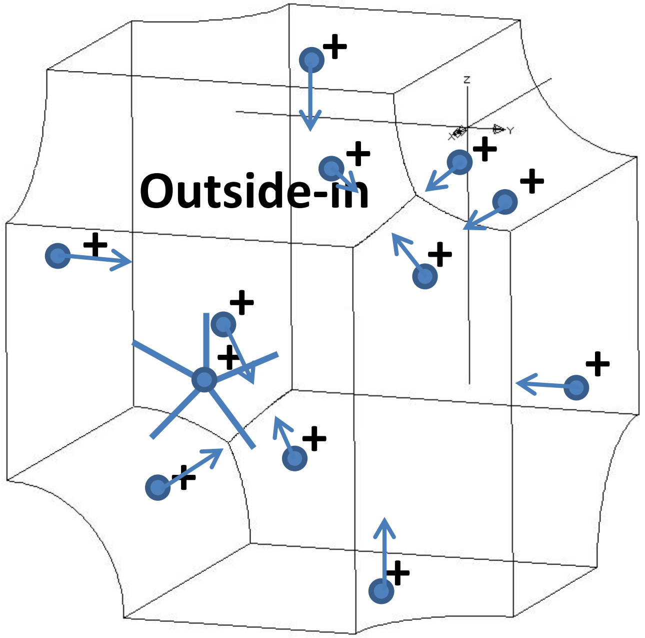
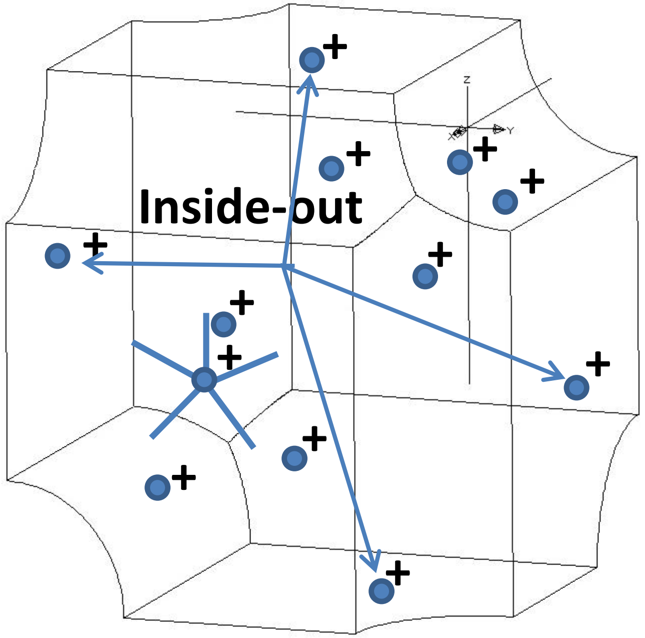




This most common axiom in hexahedral meshing is based on these assumptions:
- Specify a set of division numbers on the edges
- And/or specify an exterior quadrilateral carpet

and you will get a hexahedral mesh.

The Outside-in approach is known not to work …

The alternative axiom is based on these assumptions:
- The irregular vertices on the exterior must connect up as irregular edges throughout the volume
- When a network of irregular edges is established, a hexahedral mesh can be formed

As a consequence:
- The external quadrilateral carpet is a derived quantity
- The division numbers on the edges are a derived quantity

The Inside-out approach works again and again, no flukes …

Why call the behaviour patterns Dislocation Meshing?

The 3D Kernels

Leonard Euler's Polyhedron Formula (from 1750):

$$V - E + F = 2$$

Three-valent vertices (three faces meeting at each vertex, a particular subset of polyhedra):

$$3V = 3F_3 + 4F_4 + 5F_5$$

Two-valent edges (each edge is used by only two faces):

$$2E = 3F_3 + 4F_4 + 5F_5$$

Three types of faces (the faces with three-way, four-way and five-way irregular vertices):

$$F = F_3 + F_4 + F_5$$

When we insert these three relationships into the Euler's Polyhedron Formula, the following relationship between the faces emerges:

$$\boxed{3F_3 + 2F_4 + 1F_5 = 12}$$

The meshable shapes we want to work with will have a minimum of irregularities on the exterior, i.e., use the faces with one each of the three-way (triangle), four-way (quadrilateral) and five-way (pentagon) irregular vertices. The exterior of the polyhedra includes all of and no other than **the 2D Kernels, i.e.,** (triangle, quadrilateral and pentagon).

With this choice we exclude faces with six-way (hexagon) and more edges meeting at a vertex in the face. The subset of polyhedra will ensure that the element shape quality on the exterior will be within acceptable limits, the same starting-point as for the 2D Kernels.

We also exclude all polyhedra where more than three faces meet at any vertex to ensure that each vertex can take a single hexahedron.

The genuine 11 solutions to the equation are called **the 3D Kernels.**

Why call the behaviour patterns Dislocation Meshing?

The 3D Kernels

#	T	Q	P	V	E	F	Exists		
1	4	0	0	4	6	4	x	●	Tetrahedron
2	3	1	1	5	9	6	-		
3	3	0	3	6	12	8	-		
4	2	3	0	5	9	6	x	●	Triangular Prism
5	2	2	2	6	12	8	x	●	NoName
6	2	1	4	7	15	10	-		
7	2	0	6	8	18	12	x	●	Triangular Truncated Trapezohedron
8	1	4	1	6	12	8	-		
9	1	3	3	7	15	10	x	●	Cube with a Corner Cut Off
10	1	2	5	8	18	12	-		
11	1	1	7	9	21	14	-		
12	1	0	9	10	24	16	-		
13	0	6	0	6	12	8	x	●●	Cube, Hexahedron
14	0	5	2	7	15	10	x	●	Pentagonal Prism
15	0	4	4	8	18	12	x	●	Cube with Two Edges Cut Off
16	0	3	6	9	21	14	x	●	Cube With Three Edges Cut Off
17	0	2	8	10	24	16	x	●	Quadrilateral Truncated Trapezohedron
18	0	1	10	11	27	18	-		
19	0	0	12	12	30	20	x	● ●	Dodecahedron, Pentagonal Truncated Trapezohedron

– sequence number
T – Triangles
Q – Quadrilaterals
P – Pentagons

V – Vertices
E – Edges
F – Faces
Colour code: See next slide

In this document the notion of “k-l-m” is used to describe a polyhedron in the 3D Kernel class.
This refers to faces in the exterior separated by dashes (-):

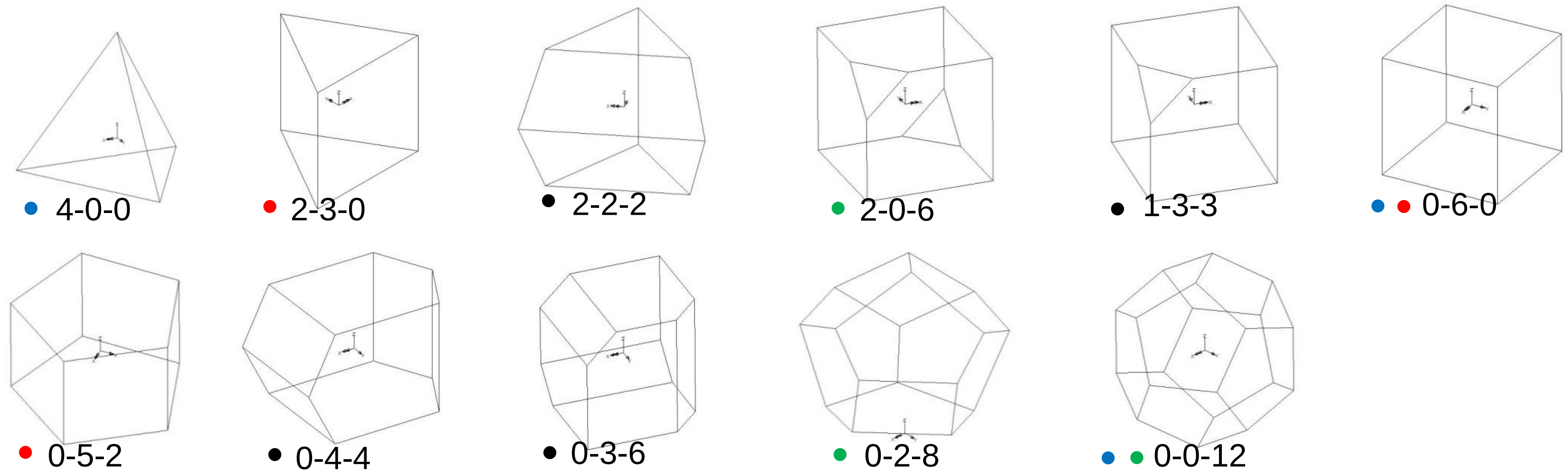
- The first digit is the number of triangles
- The second the number of quadrilaterals
- The third the number of pentagons

Hence a tetrahedron is a 4-0-0 and a cube is a 0-6-0
For additional faces, the number of hexagons are preceded with as dot (.): A truncated tetrahedron is a 4-0-0.4 . When a polyhedron has faces with higher number of faces, dashes are used to separate them.

The solutions to the scalar equation will be all the polyhedra that are bound by three-, four- and five-sided faces, i.e., have the minimum number of irregular vertices on the boundary, and meet only two at each edge and three at each vertex.
However, out of the **nineteen** arithmetic solutions, only **eleven** are genuine shapes (can be proven, proof not included here).

Why call the behaviour patterns Dislocation Meshing?

The 3D Kernels



The **3D Kernels** contains a number of familiar shapes:
- ● The three Euclidean Solids with three-valent vertices: Tetrahedron (4-0-0), Hexahedron (0-6-0) and Dodecahedron (0-0-12);
- ● The three Prisms: Triangular (2-3-0), Hexahedral (0-6-0) and Pentagonal (0-5-2) prisms;
- ● The three Truncated Trapezohedra: Triangular truncated trapezohedron (2-0-6), Quadrilateral truncated trapezohedron (0-2-8), Pentagonal truncated trapezohedron (0-0-12)

And a few that are unfamiliar:
- ● The shapes that introduce genuine 3D mesh flows in hexahedral meshes: the above and specially 2-2-2, 1-3-3, 0-4-4 and 0-3-6.

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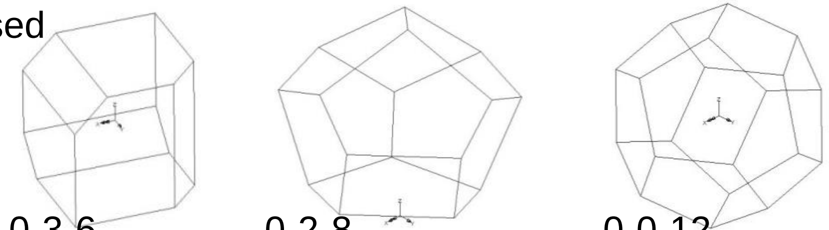
The 3D Kernels and when they are used

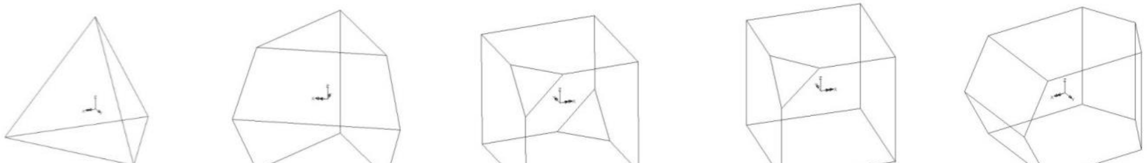
The 3D Kernels can be grouped in different ways reflecting their appearance in tessellations of other 3D Kernels.

The largest polyhedra:



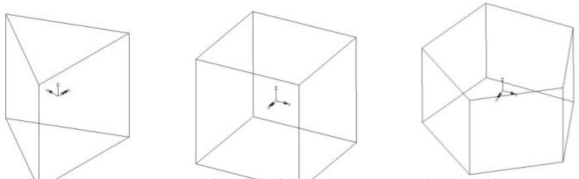
0-3-6 0-2-8 0-0-12

Intermediate Separations:



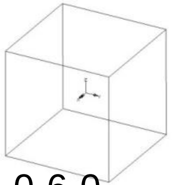
4-0-0 2-2-2 2-0-6 1-3-3 0-4-4

Full Separation:



2-3-0 0-6-0 0-5-2

Coarse Hexahedral Cluster:



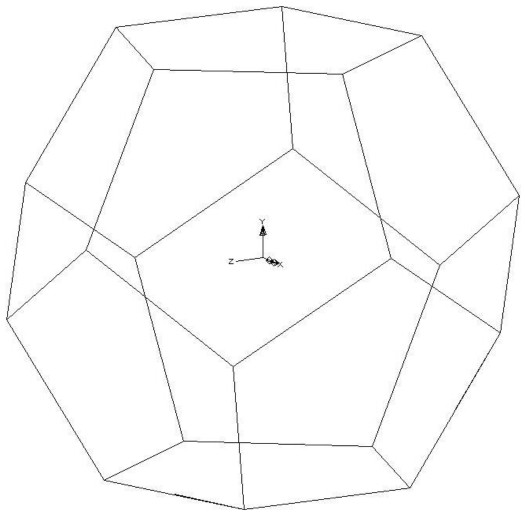
0-6-0

The largest polyhedra in the 3D Kernels can be tessellated into the other simpler polyhedra in a structured way. Simpler here means “with fewer V, E and Fs”.
These can be tessellated into other simpler polyhedra until the cluster consists of hexahedra throughout.

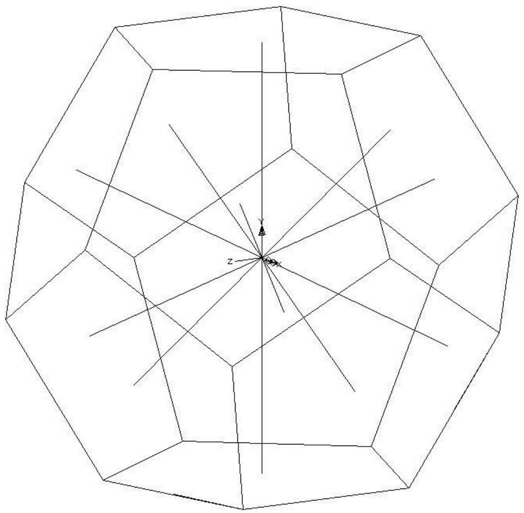
DISLOCATION MESHING –

A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

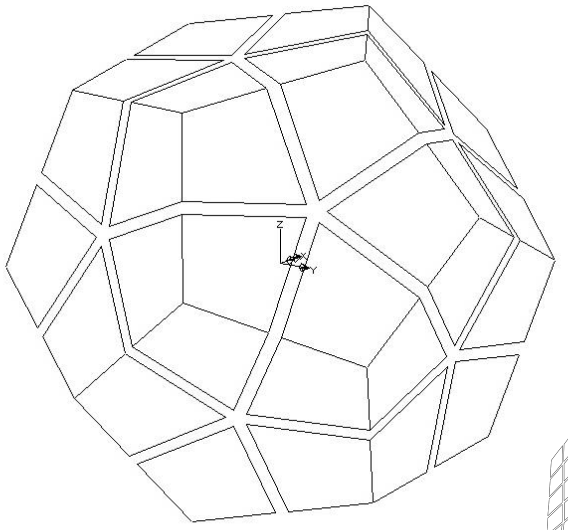
Why call the behaviour patterns Dislocation Meshing?

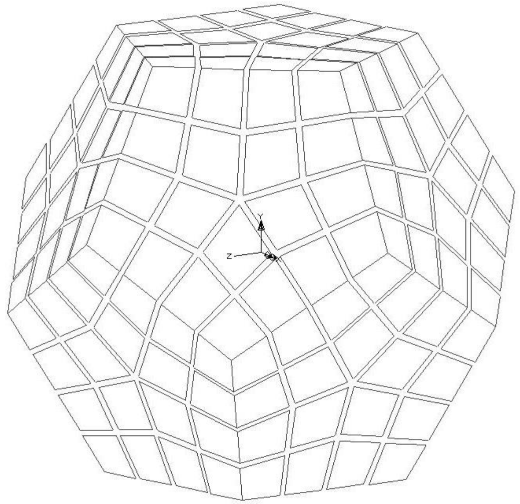


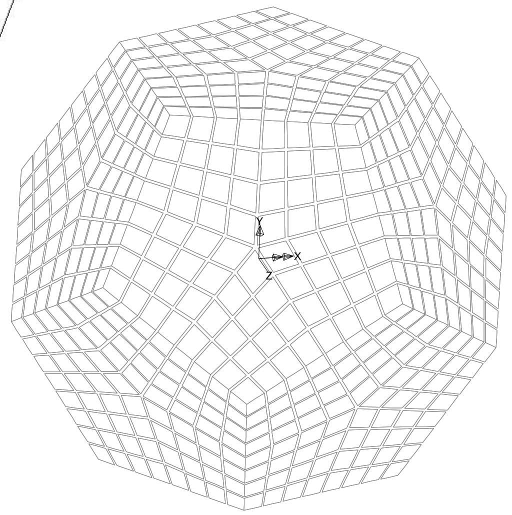
Topology visualised as a
Dodecahedron, 0-0-12.0,
the largest of the 3D Kernels



The irregular edge network for
MidPoint SubDivision (MPSD), i,e,
with no internal faces (flow sheets)



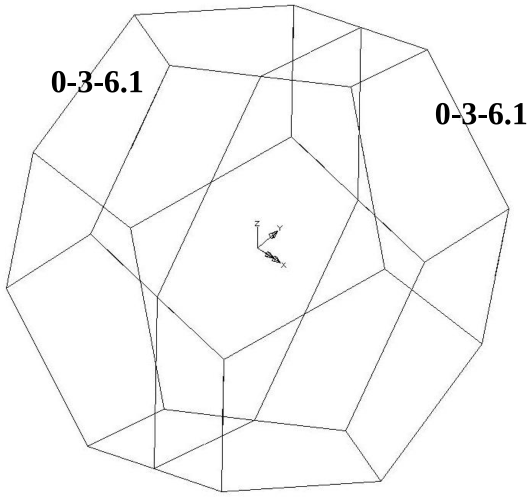




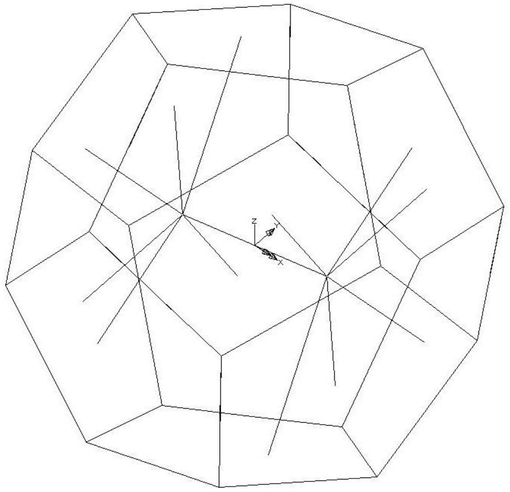
MPSD meshes with
all widths = 1
all widths = 2
all widths = 4

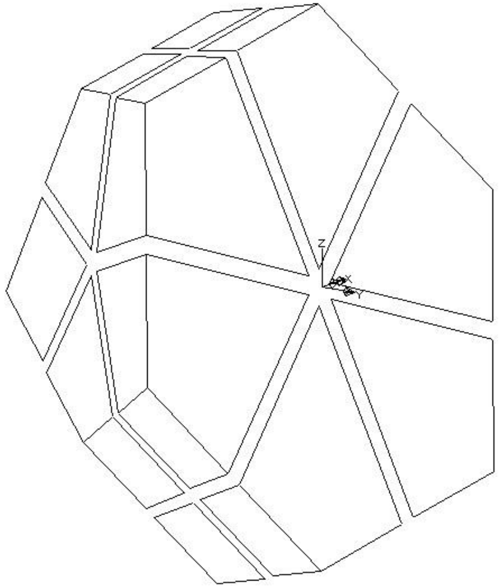
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

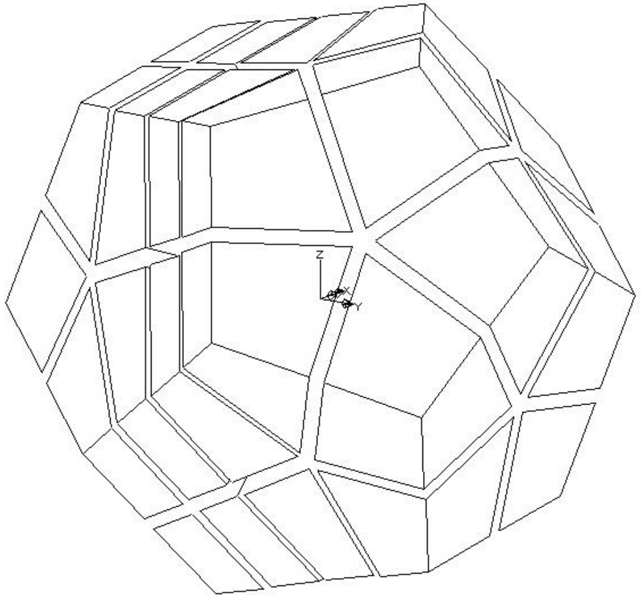
Why call the behaviour patterns Dislocation Meshing?



A cut through the object divides it into two equal parts,
not yet 3D Kernels, i.e., one internal face (flow sheet)





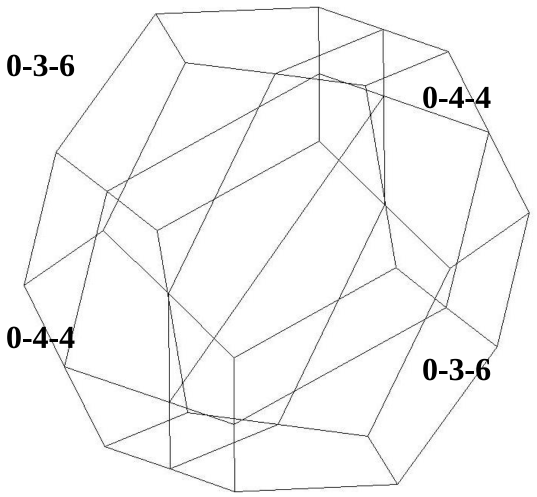


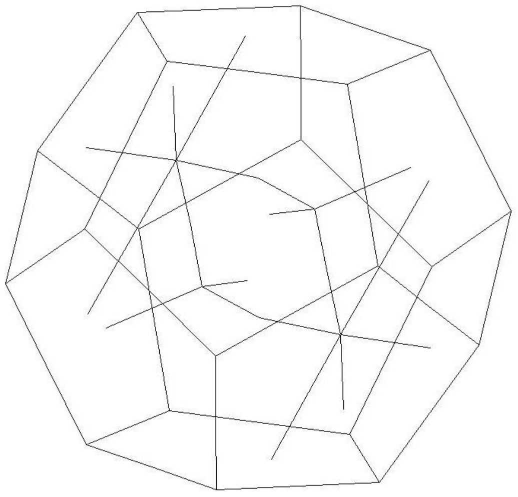
Half the mesh:
Double positive internal dislocation
vertex UNACCEPTABLE

The full mesh as a Coarse Hexahedral Cluster:
All irregular vertices are connected to a
network of irregular edges, now with one separation

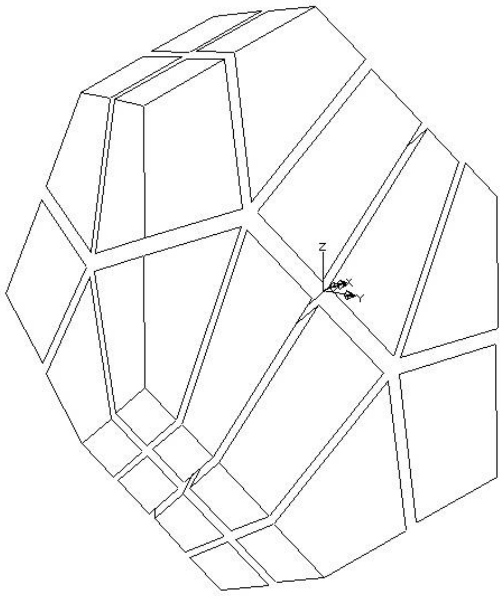
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

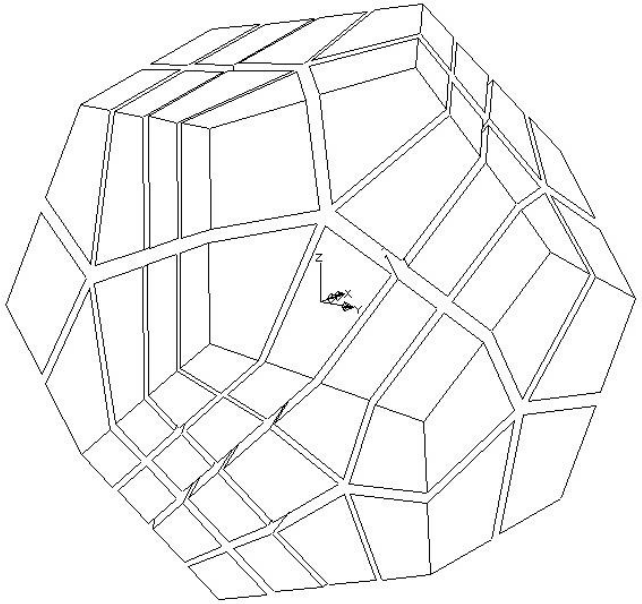
Why call the behaviour patterns Dislocation Meshing?





Two cuts through the object dividing it into four Kernels



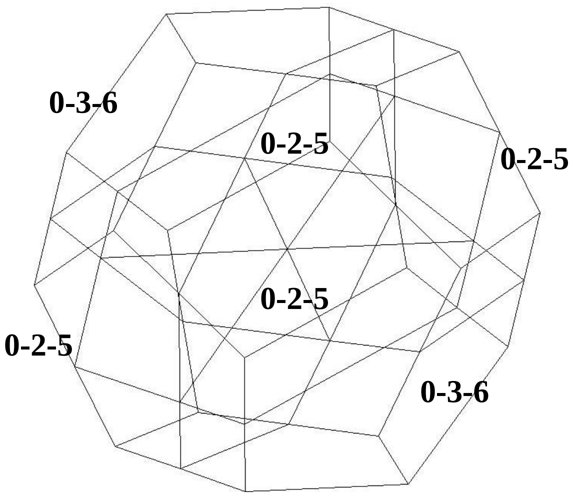


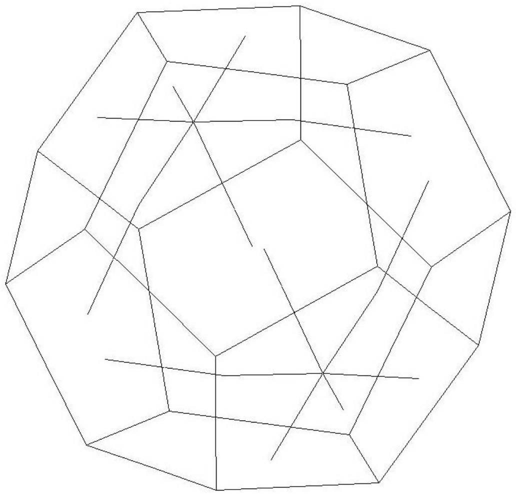
Half the mesh:
Two positive internal dislocation vertices connected, a clear IMPROVEMENT

The full mesh as a Coarse Hexahedral Cluster:
All irregular vertices are connected to a network of irregular edges, now with an increasing separation

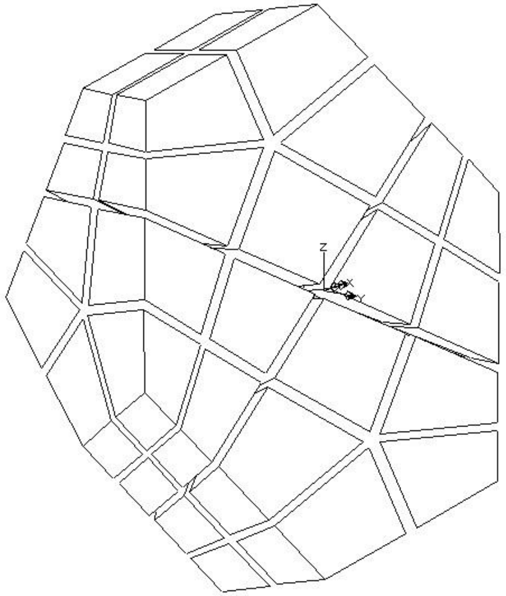
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

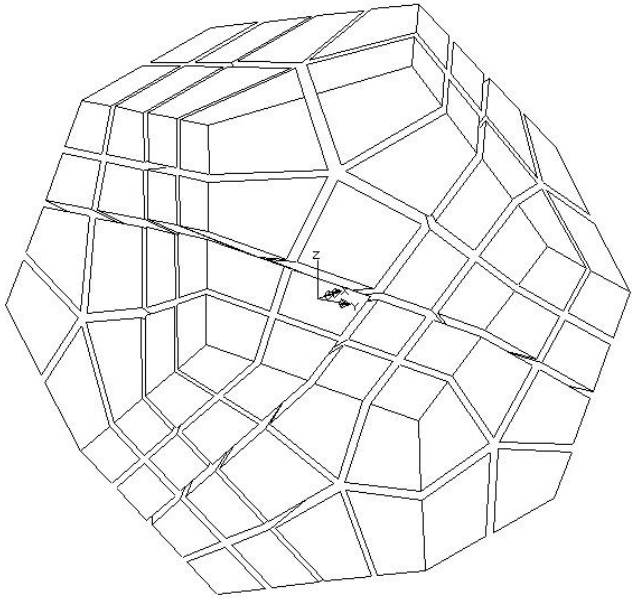
Why call the behaviour patterns Dislocation Meshing?





Three cuts through the object dividing it into eight parts



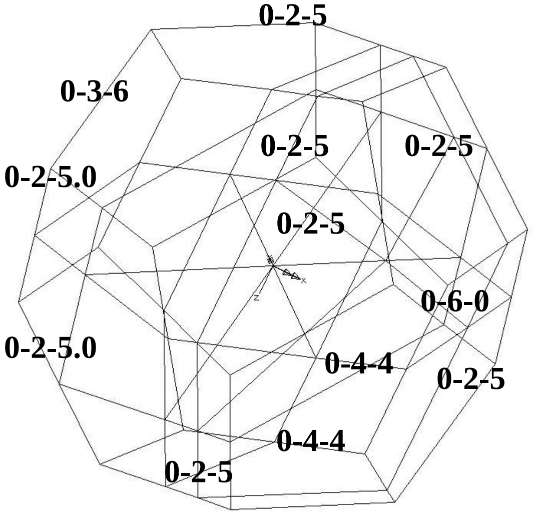


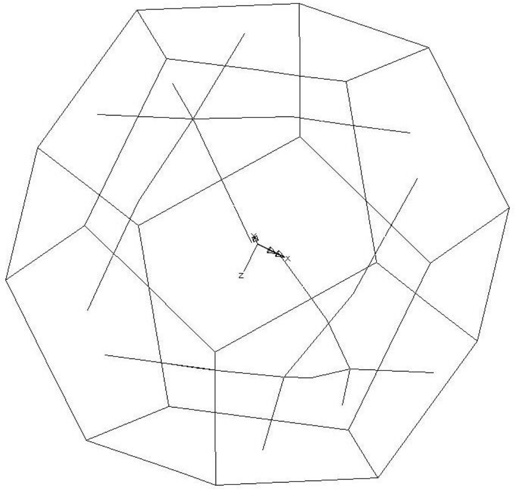
Half the mesh:
Two positive internal dislocation vertices disconnected, BETTER

The full mesh as a Coarse Hexahedral Cluster:
All irregular vertices are connected to a network of irregular edges, now with an increasing separation

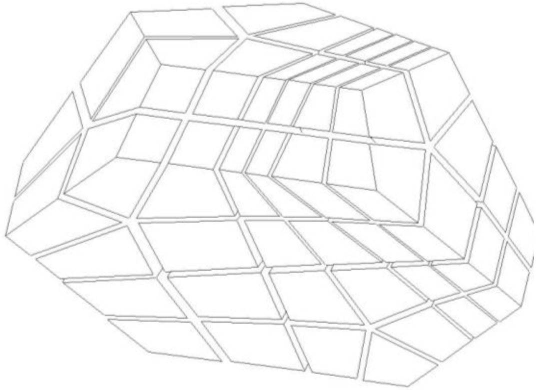
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

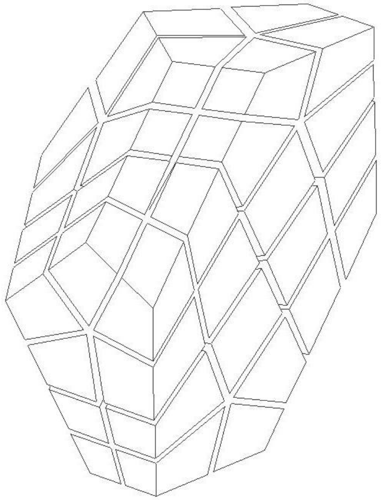
Why call the behaviour patterns Dislocation Meshing?

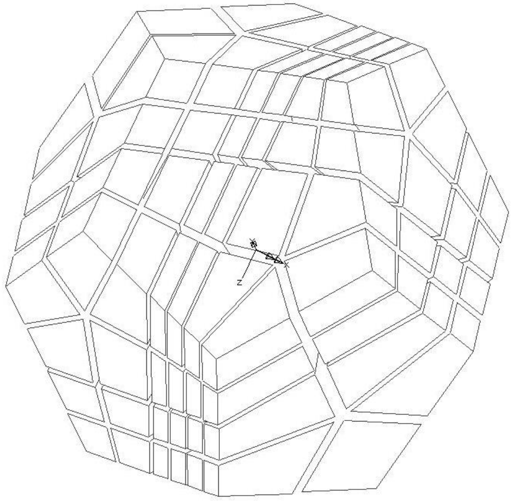




Four cuts through the object dividing it into twelve parts





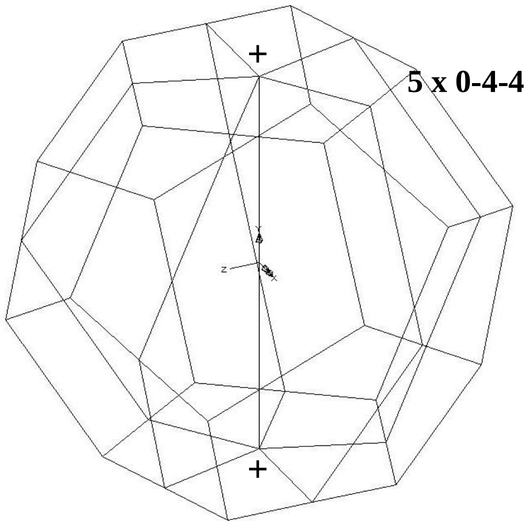


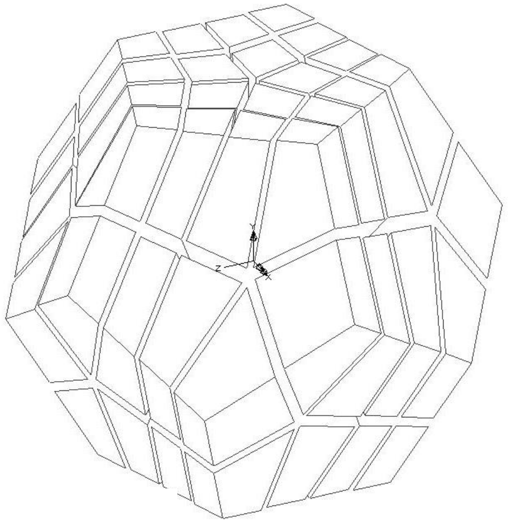
Half the mesh:
Two positive internal dislocation vertices disconnected, BETTER

The full mesh as a Coarse Hexahedral Cluster:
All irregular vertices are connected to a network of irregular edges, now with an increasing number of Prism

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

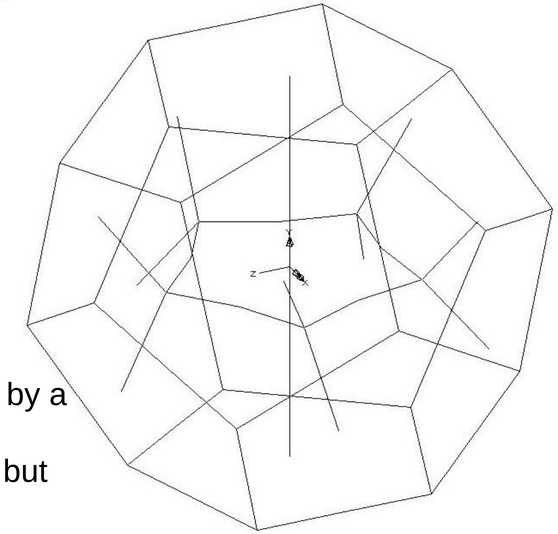
Why call the behaviour patterns Dislocation Meshing?

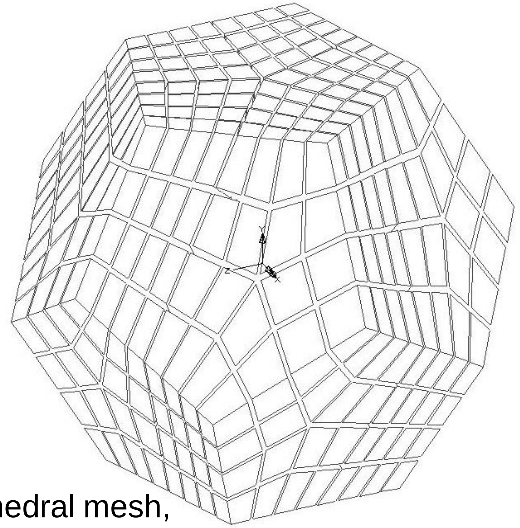




A coarse hexahedral mesh

An irregular edge cuts the
shape into five equal parts
One way … (left)



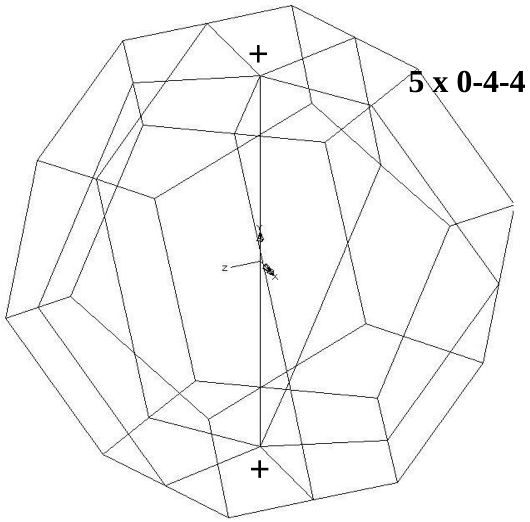


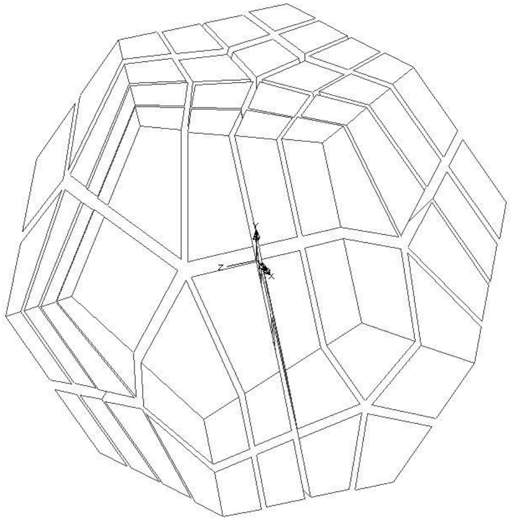
Irregular vertices are connected by a
network of irregular edges.
They cannot be fully separated, but
form a loop instead

A finer hexahedral mesh,
more of the same, only …

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

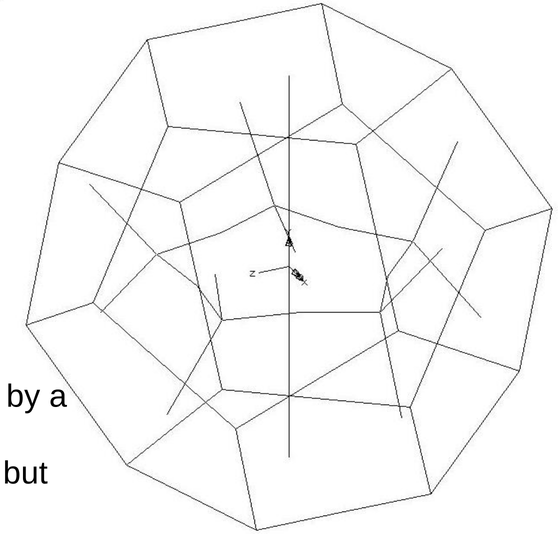
Why call the behaviour patterns Dislocation Meshing?



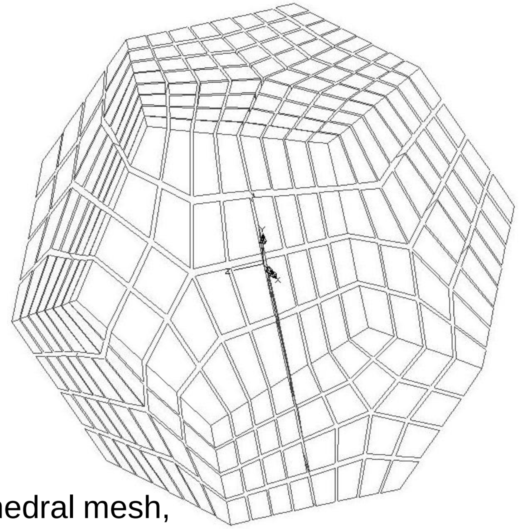


A coarse hexahedral mesh

An irregular edge cuts the
shape into five equal parts
… or the other (right)



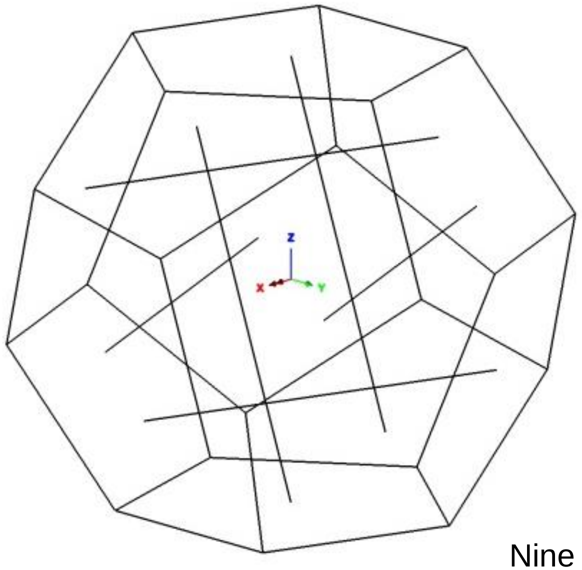
Irregular vertices are connected by a
network of irregular edges.
They cannot be fully separated, but
form a loop instead

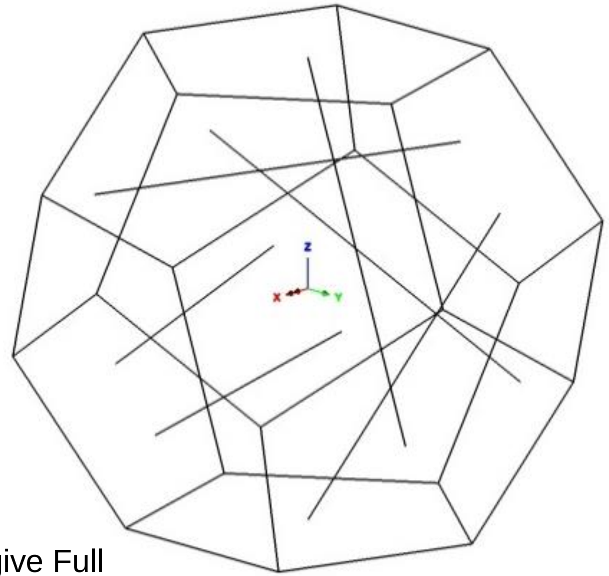


A finer hexahedral mesh,
more of the same, only …

Why call the behaviour patterns Dislocation Meshing?

Full separation





Nine internal flow sheets give Full Separation in two alternative cases

There are two alternative ways that nine internal flow sheets can be grouped to give Full Separation.

How many alternative meshes can be created using Fundamental Set for the Dodecahedron and all the groups that combine from none (MPSD) to nine (Full Separation) irregular edges?
Additionally, there are an unlimited number of meshes with the Auxiliary Set to be added.

I haven’t counted them yet, but there are many. All of them can be defined by their W solution vector.

Why call the behaviour patterns Dislocation Meshing?

In the examples for the Dodecahedron a number of cuts are introduced to create a cluster with more cells with simpler topology. The choice of cuts may seem arbitrary, but there is a structure to it:

- The chosen cuts all lead towards a fully connected cluster of Prisms that can be tessellated using the MPSD algorithm to create a Coarse Hexahedral Cluster.
- The total number of cuts to create only Prisms is nine. Then all irregular edges are separate edges, a state called Full Separation
- Any cluster created with one cut will use one of the nine alternatives.
- Any cluster created with two cuts will use any two of the nine alternatives
- Any cluster created with three cuts will use any three of the nine alternatives
- And so on …
- Hence, there are quite a few alternative ways of cutting the Dodecahedron topology into a CHC
- Additionally, there are 60 ways the twelve irregular vertices in the exterior faces can be connected forming Full Separation. And another twelve ways the top and bottom faces can be connected and the other ten irregular vertices are connected to an internal loop.

How can all these cases be kept apart? Read on, this problem has a solution …

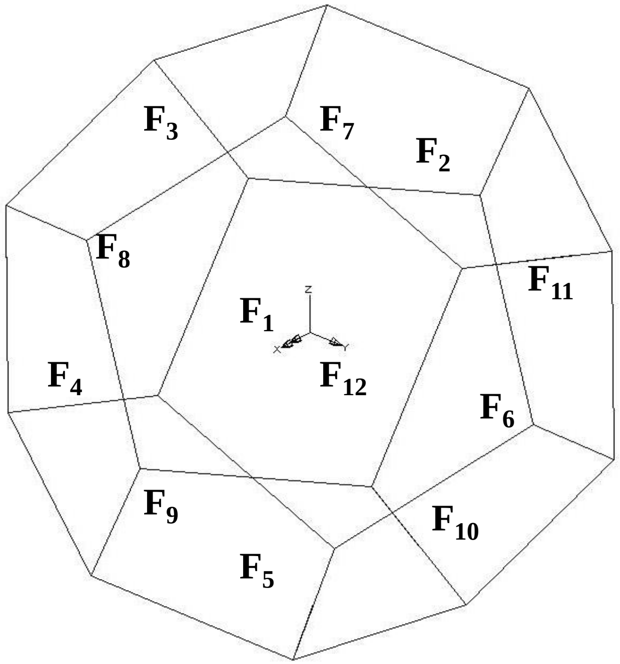
The Dodecahedron topology is used by a number of alternative geometric shapes (one example is given so far). Which of the alternative tessellations created using any number of cuts will be the “most suitable” mesh for a given geometry? A search for the “most suitable” mesh has to be conducted.
Hexahedral Meshing has in the past been a question of**: “Can you mesh this shape, Yes or No?”**
With Dislocation Meshing the question is: **“Which one to choose?”**

Well, this problem has a solution using intelligent searches …

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

Why call the behaviour patterns Dislocation Meshing?

Face Sequence Definition:



Face-Face Adjacency Matrix F_F :

$$
\begin{pmatrix}
. & 1 & 1 & 1 & 1 & 1 & . & . & . & . & . & . \\
1 & . & 1 & . & . & 1 & 1 & . & . & . & 1 & . \\
1 & 1 & . & 1 & . & . & 1 & 1 & . & . & . & . \\
1 & . & 1 & . & 1 & . & . & 1 & 1 & . & . & . \\
1 & . & . & 1 & . & 1 & . & . & 1 & 1 & . & . \\
1 & 1 & . & . & 1 & . & . & . & . & 1 & 1 & . \\
. & 1 & 1 & . & . & . & . & 1 & . & . & 1 & 1 \\
. & . & 1 & 1 & . & . & 1 & . & 1 & . & . & 1 \\
. & . & . & 1 & 1 & . & . & 1 & . & 1 & . & 1 \\
. & . & . & . & 1 & 1 & . & . & 1 & . & 1 & 1 \\
. & 1 & . & . & . & 1 & 1 & . & . & 1 & . & 1 \\
. & . & . & . & . & . & 1 & 1 & 1 & 1 & 1 & .
\end{pmatrix}
$$

In a computer implementation, each face, edge and vertex has to have an identifier, and these have to be in a sequence. The sequence will determine the identification of neighbours, i.e., the “0”s and “1”s in the matrices, and there is no limit to alternative perturbations. However, the structure they represent is invariant to the orders used.

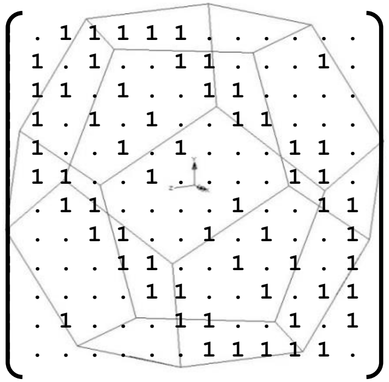
Other Adjacency Matrices used:
E_F - Edge-Face
E_E - Edge-Edge
V_F - Vertex-Face
V_E - Vertex-Edge
V_V - Vertex-Vertex

DISLOCATION MESHING –

A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

Why call the behaviour patterns Dislocation Meshing?

The Underdetermined Equation System:



```
 1  . 1 1 1 1 1 . . . . . .
 2  1 . 1 . . 1 1 . . . 1 .
 3  1 1 . 1 . . 1 1 . . . .
 4  1 . 1 . 1 . . 1 1 . . .
 5  1 . . 1 . 1 . . . 1 1 .
 6  1 1 . . 1 . . . . 1 1 .
 7  . 1 1 . . . . 1 . . 1 1
 8  . . 1 1 . . 1 . 1 . . 1
 9  . . . 1 1 . . 1 . 1 . 1
10  . . . . 1 1 . . 1 . 1 1
11  . 1 . . . 1 1 . . 1 . 1
12  . . . . . . 1 1 1 1 1 .
```

Dodecahedron 0-0-12

```
            12              40                                          30
 1  ..1..1......¦1..1..1....111.1...1...................   W -  1.............................  L = 0
 2  .1.1........¦..1.11..1.1...1.1.1....................        .1............................
 3  ..1.1.......¦.1.....1.1..11.1.1.1...................        ..1...........................
 4  ...1.1......¦1.111.1....1..1...1....................        ...1..........................
 5  .1..1.......¦.1...1.1111.....11.....................        ....1.........................
 6  1.....1.....¦.....................11.1.11.111.......        .....1........................
 7  1.........1.¦....................1..1.11111.1.......        ......1.......................
 8  ..1.......1.¦1..1..1......1......1..1.1..1..........        .......1......................
 9  .....11.....¦...........11..1...1.11.1.....1........        ........1.....................
10  1......1....¦.....................11.1....1..1111....        .........1....................
11  .1.....1....¦..1.1...........1.1.............1111....        ..........1...................
12  ...1..1.....¦.....1..1.1...1...........11..11.......        ...........1..................
13  1.......1...¦....................1...1.......111.111.        ............1.................
14  ..1.....1...¦.1.....1.1.........11...............111.        .............1................
15  ....1..1....¦............11.1.1...11......1.....1....        ..............1...............
16  1........1..¦....................1..1.11.......1.11.1        ...............1..............
17  ...1.....1..¦...11.1....1...........1.11............1        ................1.............
18  .....1..1...¦1.1...........1...1.....1.......11....1.        .................1............
19  ....1.....1.¦.....1.1..1.....1..........111.1.......        ..................1...........
20  .1........1..¦.1......11.......1................1.11.1        ...................1..........
21  ..1........1¦.1....1.............1....1..1.......1.11        ....................1.........
22  .1.........1¦.1.................1.............1.111.11        .....................1........
23  .......1..1.¦1.111........1..1......1.........1......        ......................1.......
24  ...1.......1¦......1...1...............1111.1........1        .......................1......
25  ......1.1...¦.....1.111....1....1............1.....1..        ........................1.....
26  ....1......1¦..........1....1......1....1111....1....        .........................1....
27  .......1.1..¦...11......111...1...1.1...............        ..........................1...
28  .....1.....1¦...............1..1...1.1.....1.1..1..1.        ...........................1..
29  ........1.1.¦1.1..1.1......1.1...............1.1......        ............................1.
30  ......1..1..¦.........11.11....1.1.1...............1..        .............................1
```

All the irregular edge networks and meshes shown for the Dodecahedron so far are represented by this single equation system and its G-matrix.

This is the subset of hexahedral meshes defined as the Fundamental Set

Group: 001

…

Group: 072

```
 1   001  111.1....1.........1.....11......1..
 2   002  111.........1.....11.....1.....1..1
 3   003  111.1.............1.....11....11..
 4   004  111.....1......1......11..1..1.....
 5   005  111.1....1...............1.11......1..
 6   006  111.1...............1.11....11..
 7   007  .1.11..1.1..........111....1........
 8   008  11.11....1.........1.....11......1..
 9   009  11.11....1...............1.11......1..
10   010  11.11.................1.11....11..
11   011  .1.11......1...............1.1.1....1.1.
12   012  11.11.................1.....11....11..
.... Cont ...
```

Due to the size of the G-matrix for a 0-0-12, only the first 12 groups are included here.

Why call the behaviour patterns Dislocation Meshing?

An F_F matrix can be transformed into an underdetermined equation system that represent all the ways the irregular edge networks in the volume can connect the irregular vertices in the exterior. The minimum number of irregular vertices in the exterior defines the Fundamental Set.

$$\mathbf{F_F} \Rightarrow \mathbf{Aw = L}$$

$$\underset{\left(\mathbf{G}\right)}{\left(\mathbf{A_e \mid A_i}\right)}\begin{pmatrix}\mathbf{w_e}\\ \mathbf{w_i}\end{pmatrix} - \left(\mathbf{I}\right)\left(\mathbf{L}\right) = \mathbf{0}$$

This equation system relates the widths of the flow sheets in the topology with the length of the external edges.

It contains all the possible flow sheets for all the different combinations of free dislocation edges

Where:
A_e – is the Edge-Face (External Flow Sheet) adjacency matrix for the topology
A_i – is the Edge- Internal Flow Sheet adjacency matrix for the topology
w_e – is the External Flow Sheet Width vector for the topology
w_i – is the Internal Flow Sheet vector for the topology
I – is the identity matrix (“1” on the diagonal, “0” everywhere else)
L – is the edge length (division number) vector for the topology
G – is the matrix that identifies which internal flow sheets form a group

The A-matrix defines the external and all the internal flow sheets that are used by all the possible groups of irregular edge networks for this topology. The G-matrix identifies which of the internal flow sheets that together will tessellate the polyhedron into a cluster of Prism, i.e., Full Separation for the irregular edges.

Why call the behaviour patterns Dislocation Meshing?

How to solve the Underdetermined Super Set Equation System

$$\underbrace{\begin{bmatrix} \mathbf{A_e} \mid \mathbf{A_i} \end{bmatrix}}_{\begin{bmatrix} \mathbf{G} \end{bmatrix}} \begin{bmatrix} \mathbf{w_e} \\ \mathbf{w_i} \end{bmatrix} - \begin{bmatrix} \mathbf{I} \end{bmatrix}\begin{bmatrix} \mathbf{L} \end{bmatrix} = 0$$

A large number of cases are compressed into the equation system. Each of them can be defined uniquely by:
- The solution vector expressed as a vector of Widths for both the external and the internal flow sheets.
- The equation system itself.

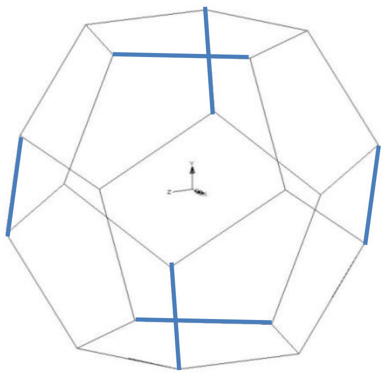
To create a square equation system that can be solved, a number of unknowns must be specified, i.e., Fixed, both which they are and their values. The remaining unknows are Free, but a function of the Fixed unknowns,

- The G-matrix describes which internal flow sheets belong in a group, i.e., that between them can tessellate the shape into a cluster of Prism with Full Separation of the irregular edges. By specifying which internal flow sheets should be present (“1”) and which should be absent (“0”) in a Group, the cluster of cells will change accordingly. When an internal flow sheet is declared absent, all occurrences down the G—matrix are affected, simplifying the assessment of the remaining alternatives.
- The introduction of both Widths and Lengths on the left-hand side of the equation system opens up for the specification of:
    - All the fixed unknown as Lengths;
    - All the fixed unknowns as Widths;
    - All the fixed unknowns as a mix of Lengths and Widths.

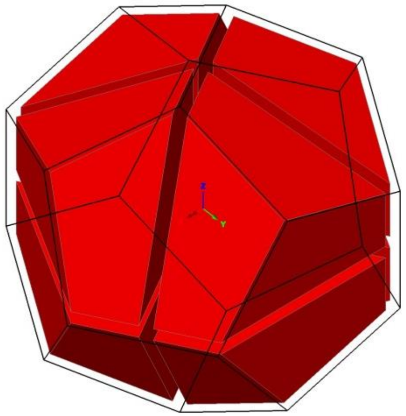
Algorithms for choosing good combinations of Fixed unknowns has to be developed using Deep Learning Technology.

What to use it for: A Worked Example, Tessellation and Meshing

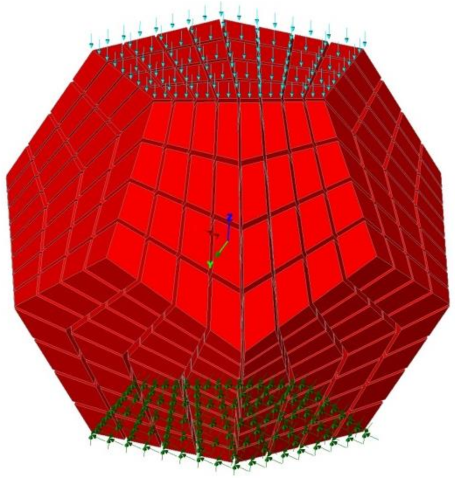
A diversion: What is the 0-0-12 hexahedral mesh with the fewest number of elements?



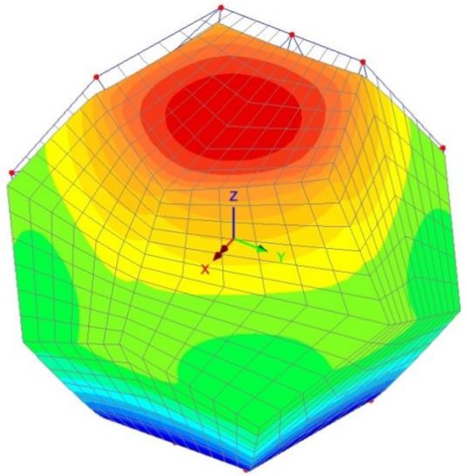
Positions for all six irregular edges for minimum number of elements solution



All external flow sheet widths = 0
CHC gives eight cells and elements



The remaining internal flow sheets widths = 4 with loads and boundary conditions shown



The deformation contours for internal flow sheets widths = 6

What is the 0-0-12 hexahedral mesh with the fewest number of elements?

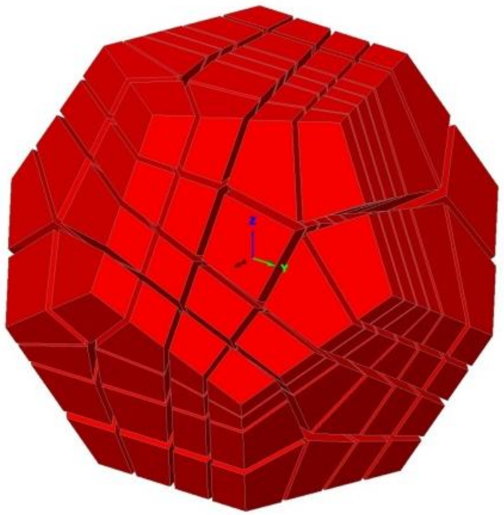
What about the case where all the irregular edges are separated (Full Separation) and all placed in an edge, a symmetrical case? This is described with the width of all external flow sheets are null (0), and the irregular edges are all in edges on the boundary. The number of cells is reduced to eight. When each cell is meshed with one element each (CHC), the minimum number of elements is eight.

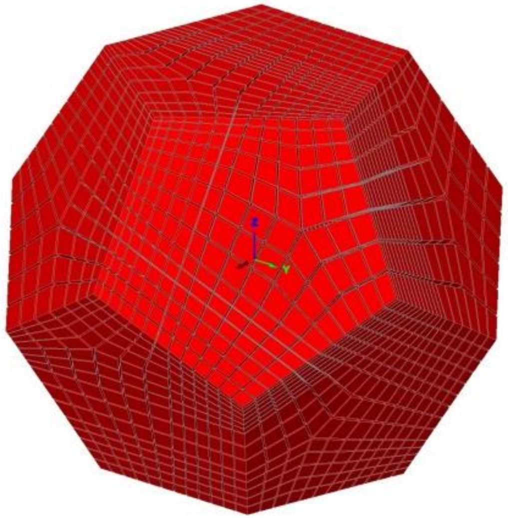
Any other with fewer elements?

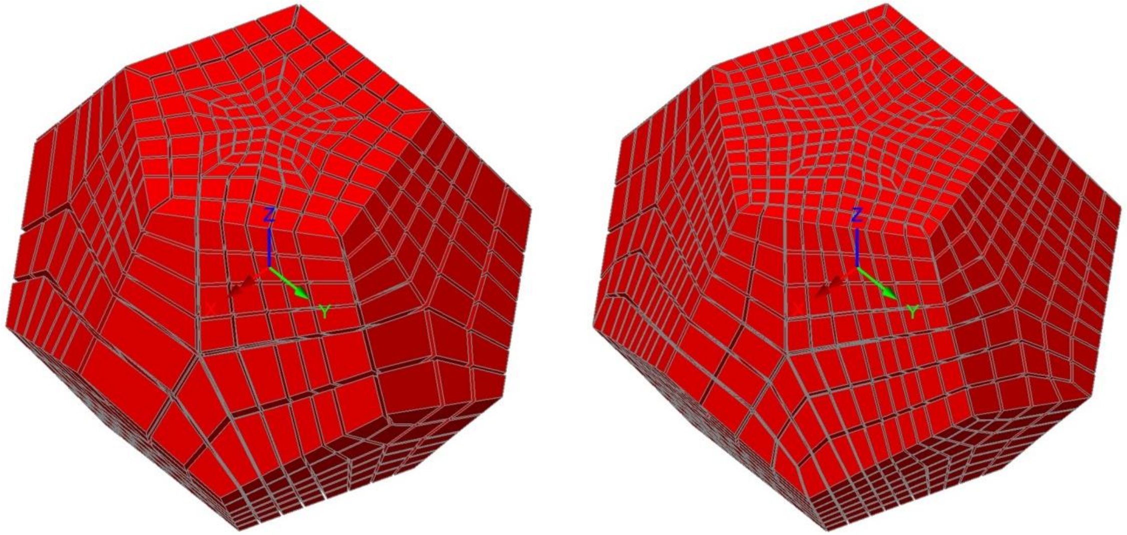
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: A Worked Example, Tessellation and Meshing

Examples of meshes where Fundamental Set, Regular and Transitional Meshing are used







A mesh where the Fundamental Set is created using flow sheets, here creating a CHC.

A mesh where the Fundamental Set is used and Regular Hex-in-Hex Meshing creates a finer mesh.

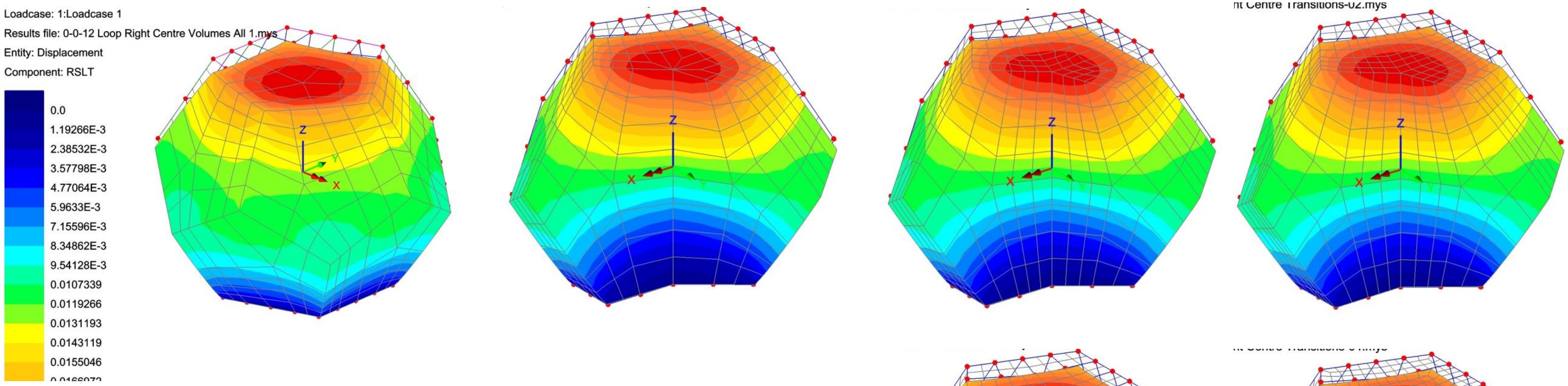
Examples of a mesh where the Fundamental Set is used, and Regular and Transitional Hex-in-Hex meshing is used.

The irregular edge networks that depends on the topology of the shape must create the Fundamental Set of flow sheets and edges. The end-result will be the Coarse Hexahedral Cluster. Regular Hex-in-Hex meshing creates a regular mesh for h-convergence studies.
However, additional irregular edge networks can be added in pairs using the Auxiliary Set. The irregular edges in the Auxiliary Set are independent of the topology of the shape, and can be added in any number.
For practical reasons, in Dislocation Meshing the Auxiliary Set is introduced using Transitional Hex-in-Hex meshing on a Coarse Hexahedral Cluster and used repeatedly on Hexahedra coming out of the Hex-in-Hex meshing processes.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

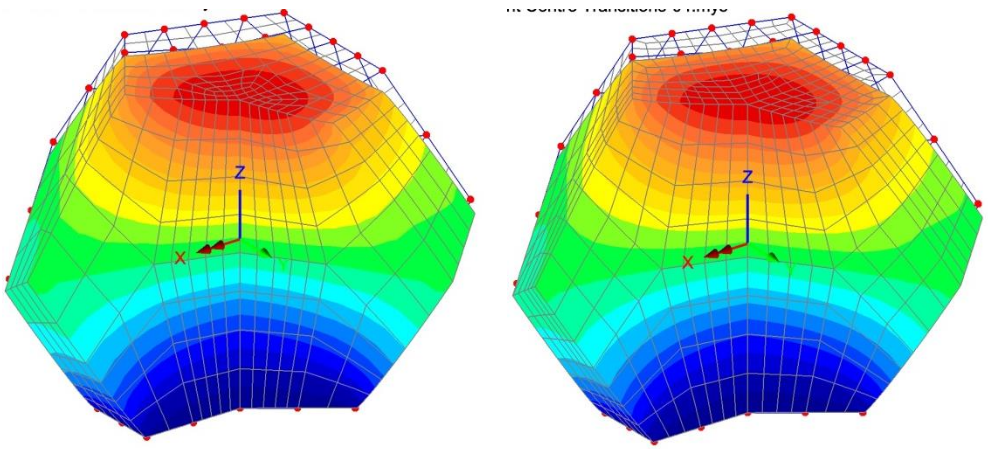
What to use it for: A Worked Example, Tessellation and Meshing

Examples of meshes where Fundamental Set, Regular and Transitional Meshing is used



Examples of results from meshes where Fundamental, Regular and Transitional Meshing is used. The Dodecahedron is first Tessellated as a pentagonal truncated trapezohedron case with Right orientation of the irregular edge network and meshes with all width = 2. The Regular Hex-in-Hex meshing is then replaced by the Transitional Hex-in-Hex meshing giving significant increase in the number of elements in the core of the mesh. The results are cut through showing the inside of the mesh.

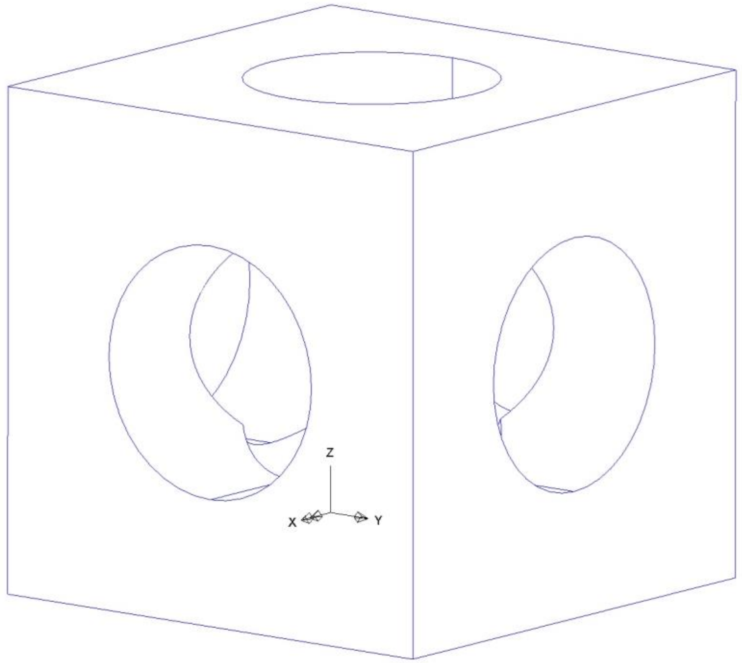
For this load case / boundary condition combination, the results are near identical for varying meshing methods and mesh densities.



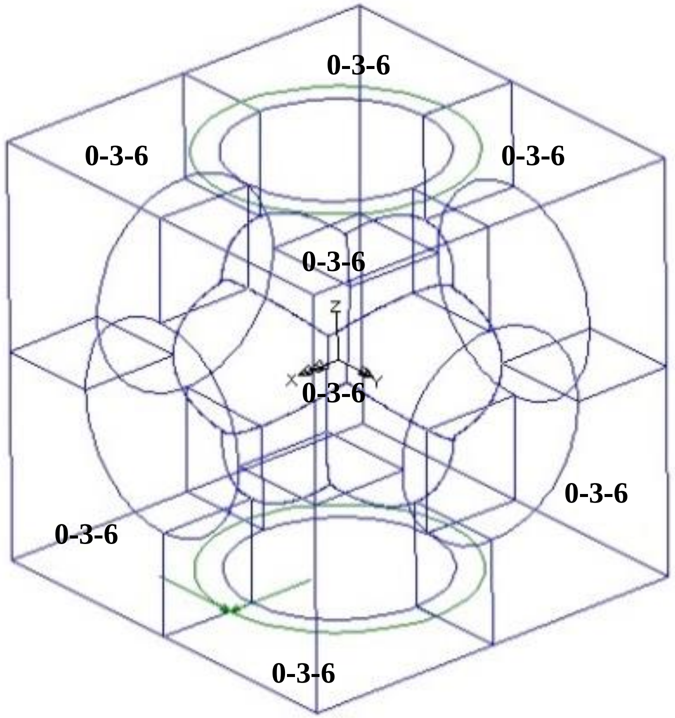
DISLOCATION MESHING –

A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

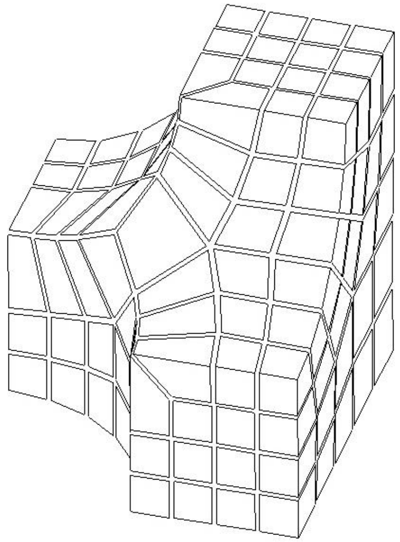
What to use it for: A Worked Example, Tessellation and Meshing



A solid model, a cube with three intersecting holes
With the same diameter



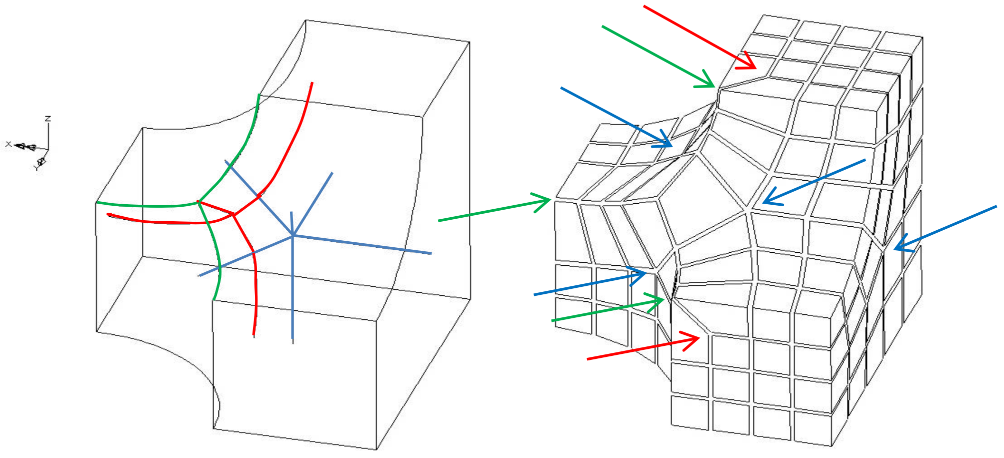
A cluster of 3D Kernels, 8 x (0-3-6)



A sample mesh of 1/8th of the shape

The shape is a cube with three intersecting holes going through it between pairs of opposing faces. The first step in the tessellation process creates eight 0-3-6s and each of them is tessellated to a Coarse Hexahedral Cluster using a chosen irregular edge network. There are a number of alternative irregular edge networks,

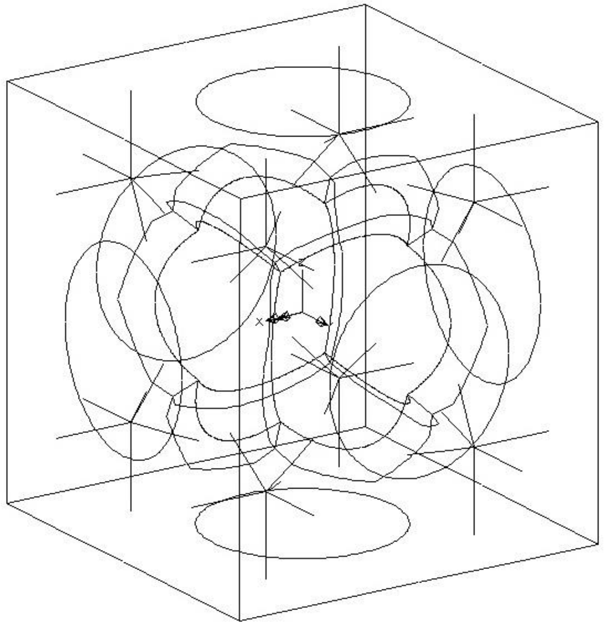
What to use it for: Examples, A Structured Mesh



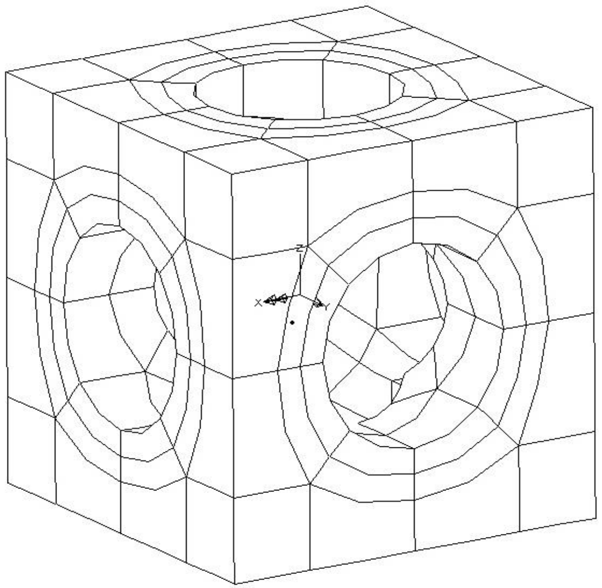
A 1/8th of the shape and the chosen irregular networks and mesh

The chosen irregular edge networks are:

- The Fundamental Set network for an 0-3-6 with only five-way irregular edges (in blue) and no internal flow sheets (MPSD)
- the Auxiliary Set networks for the transitional mesh (three-way irregular edges in red, five-way irregular edges in green).



The assembled irregular edge networks



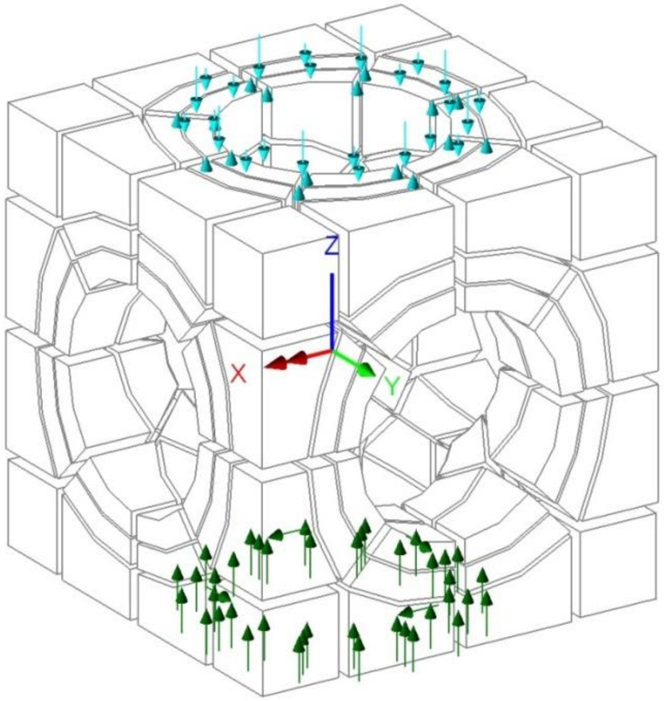
The corresponding Coarse Hexahedral Cluster

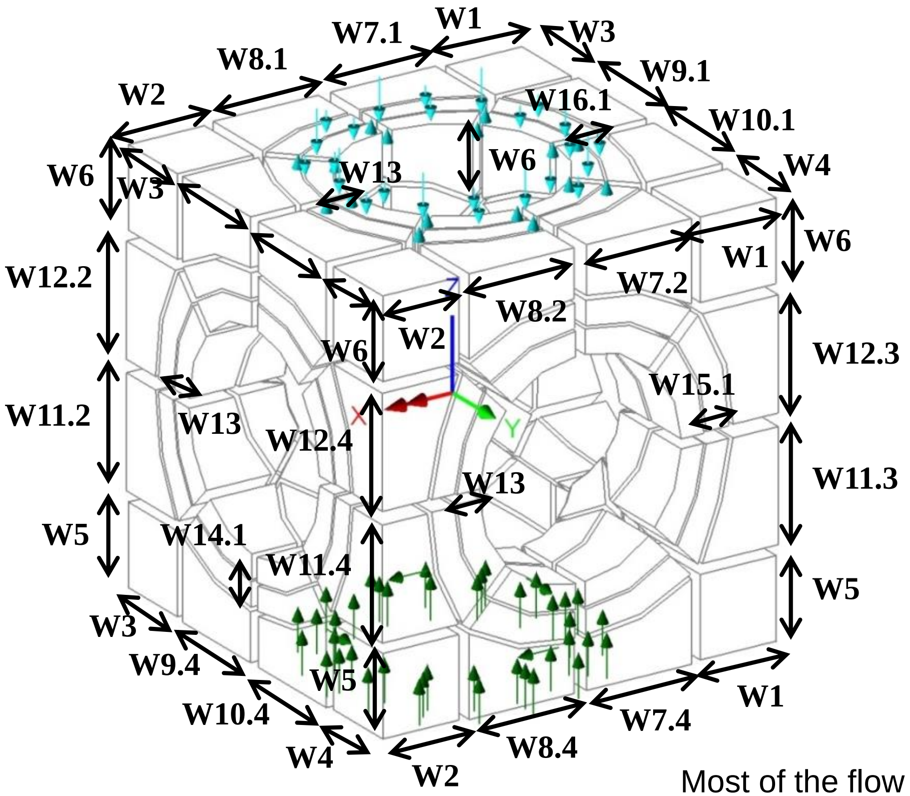
The MPSD network connects the external irregular vertices into one mid-volume irregular vertex, no internal flow sheets are used. This works well here as the 1/8th has the same dimensions in 3 directions. The irregular edge pair is added due to the concave internal vertex as the intersection point of the three cylinders.
The assembled irregular edge network for the cluster can then be used as Volume Mesh Controls to a meshing process that creates a Coarse Hexahedral Cluster.

DISLOCATION MESHING –

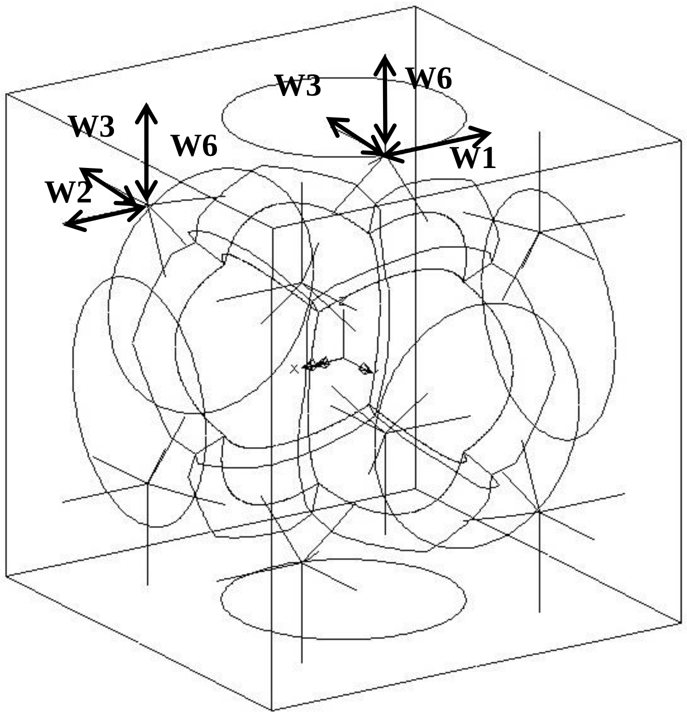
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, A Structured Mesh





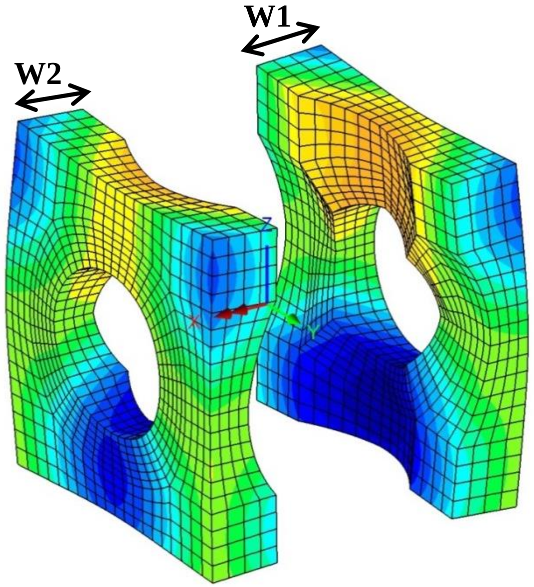
Their corresponding Coarse Hexahedral Cluster



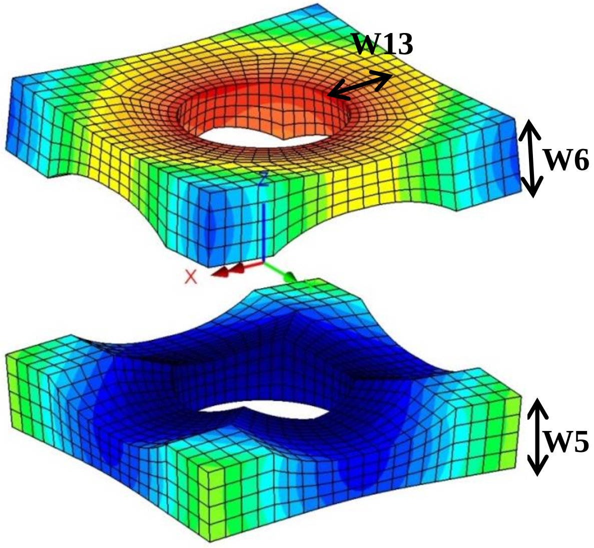
Most of the flow sheets annotated

The resulting Coarse Hexahedral Cluster has 37 flow sheets, all with a width of “1” (one). The model can be augmented with loads, boundary conditions and material properties and a linear static analysis can be run.
(Yes, the load and boundary conditions are simplistic, but they give nice looking deformation contour plots and it proves a point …).
The flow sheets in this model are explored further in the next few slides.

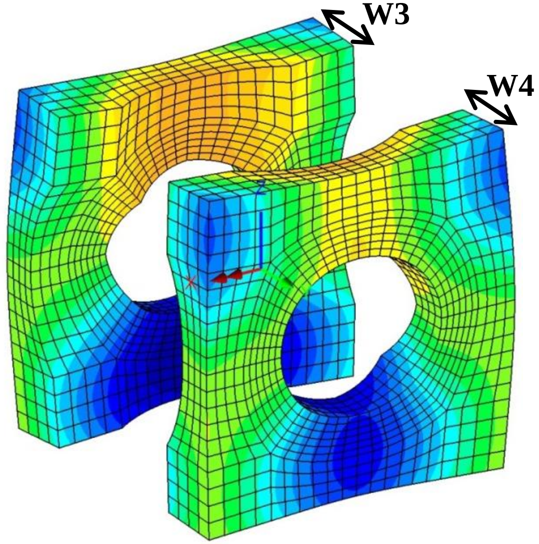
What to use it for: Examples, A Structured Mesh



Flow sheets W1&W2



Flow sheets W5, W6 and W13



Flow sheets W3&W4

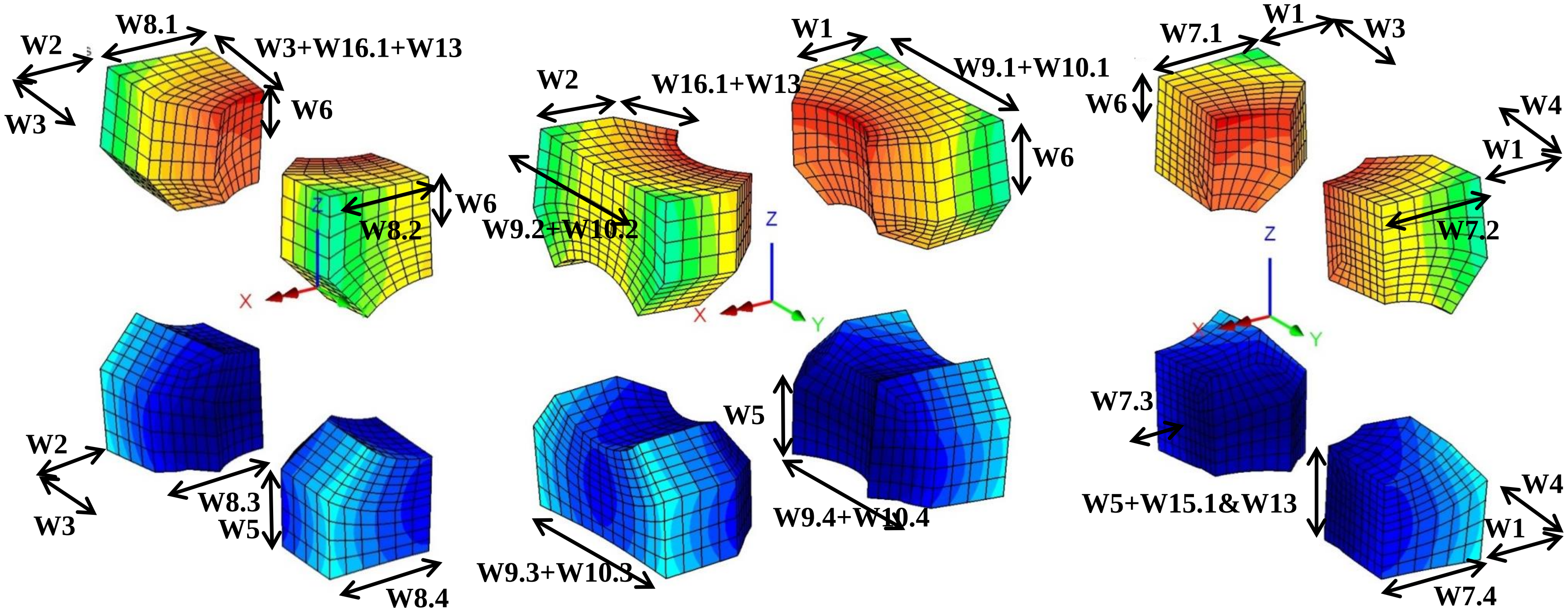
The exploration of external and internal flow sheets uses pictures with contours from a linear static analysis. The contours are deformation contours on a deformed mesh, to demonstrate that this is a fully working hexahedral mesh that can be used for analysis.

The following pictures show the external and internal flow sheets for a mesh with widths set to four for most flow sheets.

For this set of irregular edge networks, there are 37 independent flow sheets:

- There are 7 external flow sheets: W1,W2,W3,W4,W5,W6 and W13 as shown above;
- There are 30 internal flow sheets, all the rest …

What to use it for: Examples, A Structured Mesh

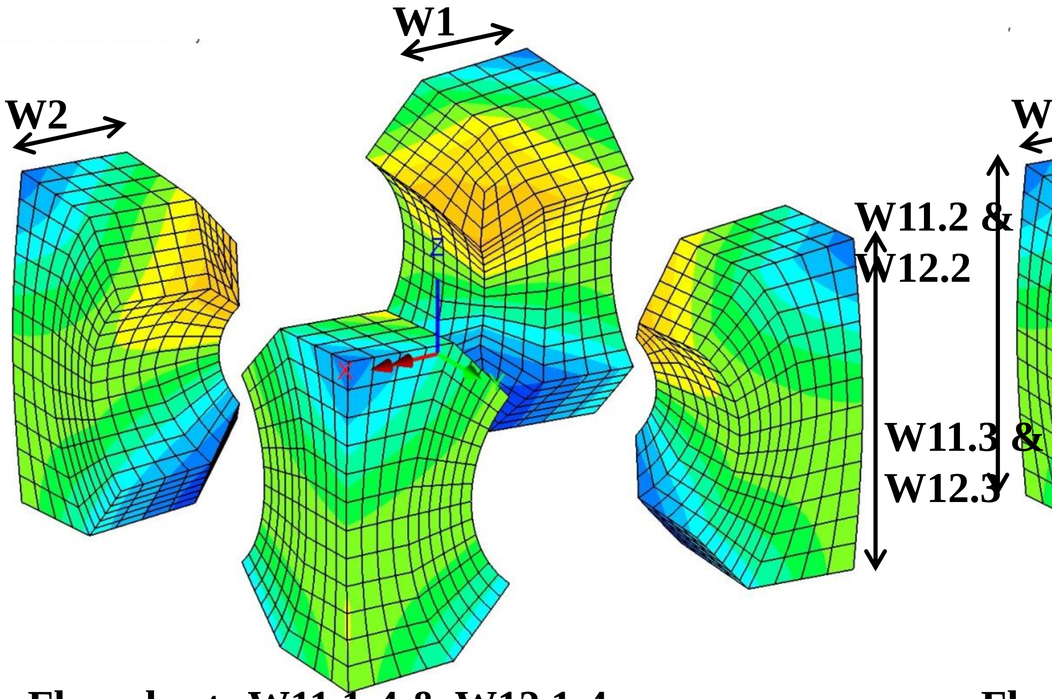


Flow sheets W8.1-W8.4 | Flow sheets W9.1-W9.4 & W10.1-W10.4 | Flow sheets W7.1-W7.4

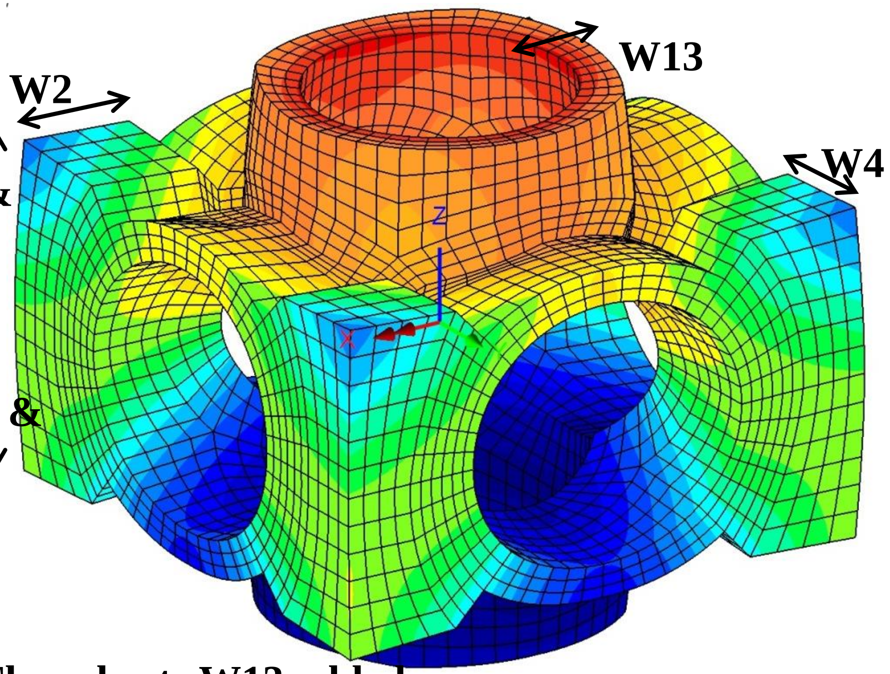
A flow sheet is independent of any other flow sheet in the model. The width can be defined to “blend in” with the neighbours, so the mesh is graded across the boundaries between cells. Each cell has three flow sheet crossing each other within, giving the freedom to define any mesh density independently in each of three directions.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, A Structured Mesh



Flow sheets W11.1-4 & W12.1-4



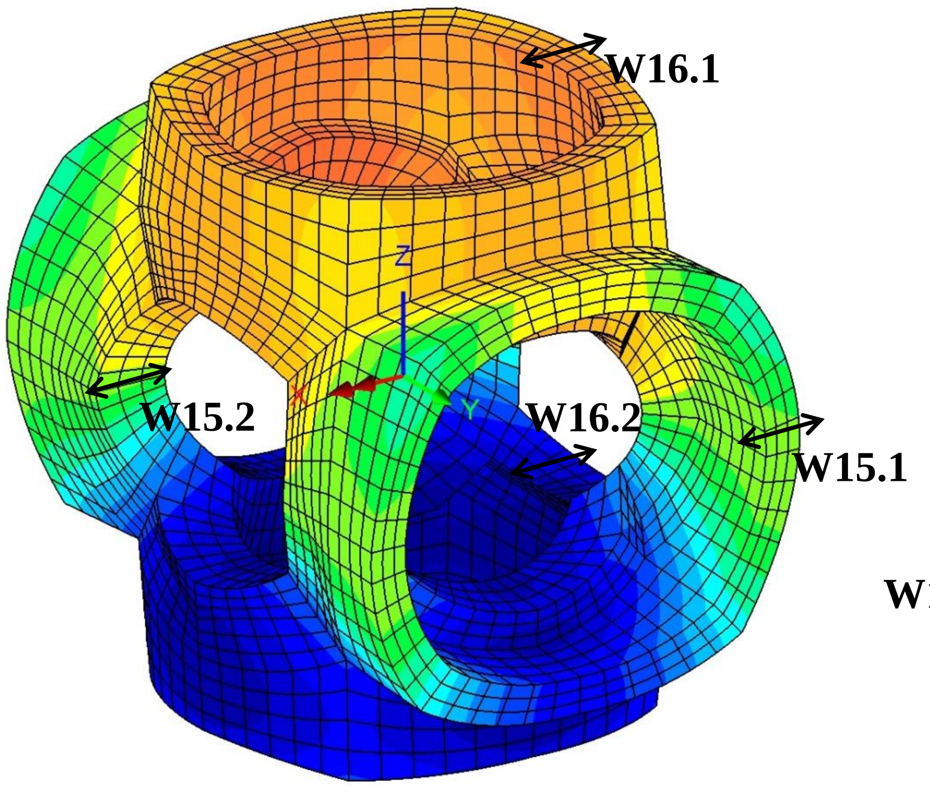
Flow sheets W13 added

To create a well-formed mesh, the widths have to be chosen so all elements are close to cubes, avoiding poorly shaped elements and warnings during the meshing process or during the analysis. Hence, there are constraints on the range of widths for crossing flow sheets.

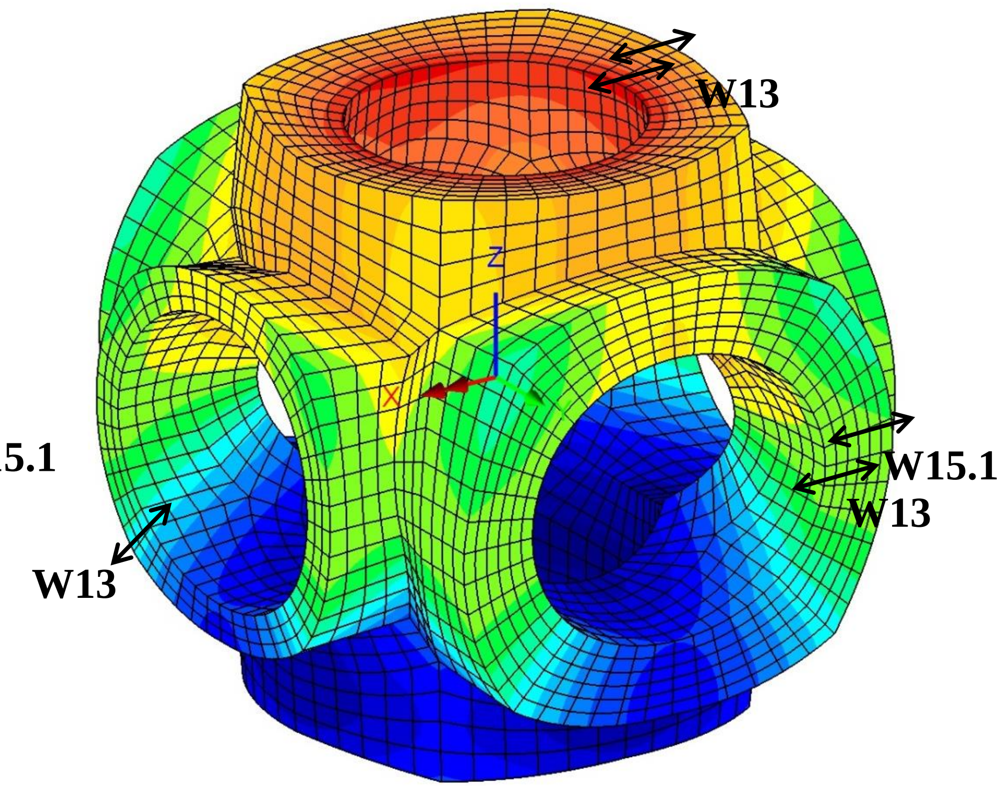
As the element size decreases, the number of elements goes up dramatically, requiring significantly increased computer resources for the analysis. However, the element shape quality in the mesh is unaffected.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, A Structured Mesh



Flow sheets W15.1&W15.2, W16.1 & W16.2

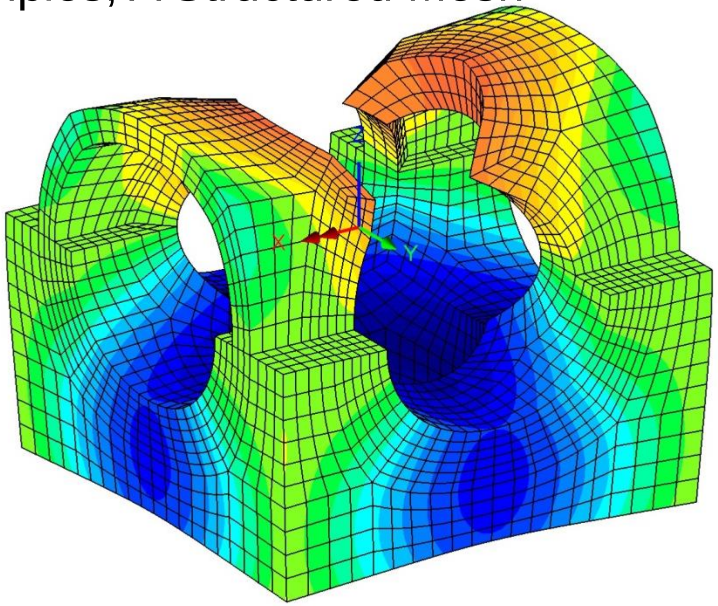


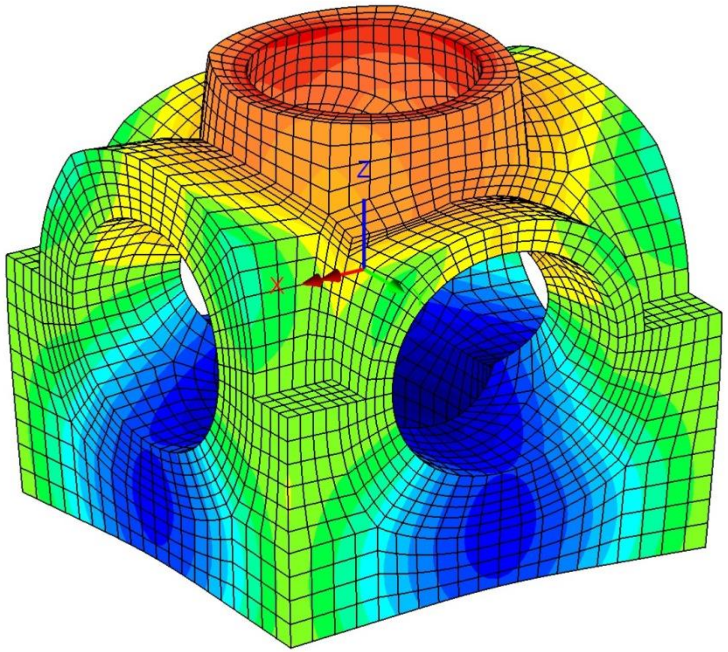
Flow sheet W13 added

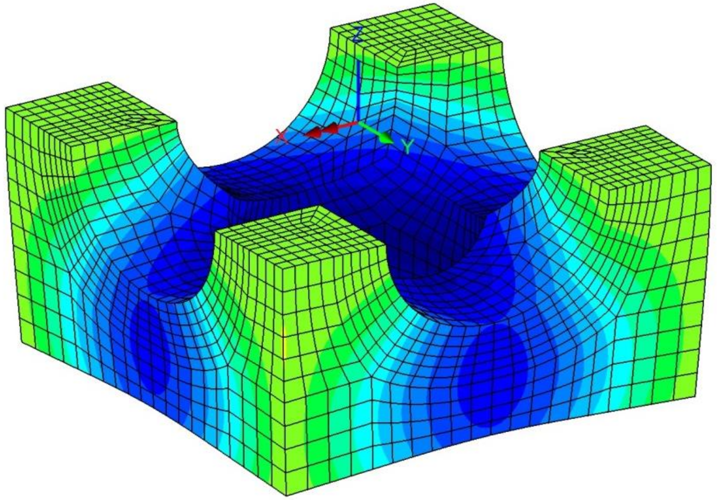
The Volume Mesh Controls include the definition of the flow sheets:

- their presence or absence (“1” or “0”) of the internal flow sheets, i.e., their positioning relative to other irregular edge networks in the cells
- the width of each flow sheet, giving absolute position for the irregular edge networks.

What to use it for: Examples, A Structured Mesh

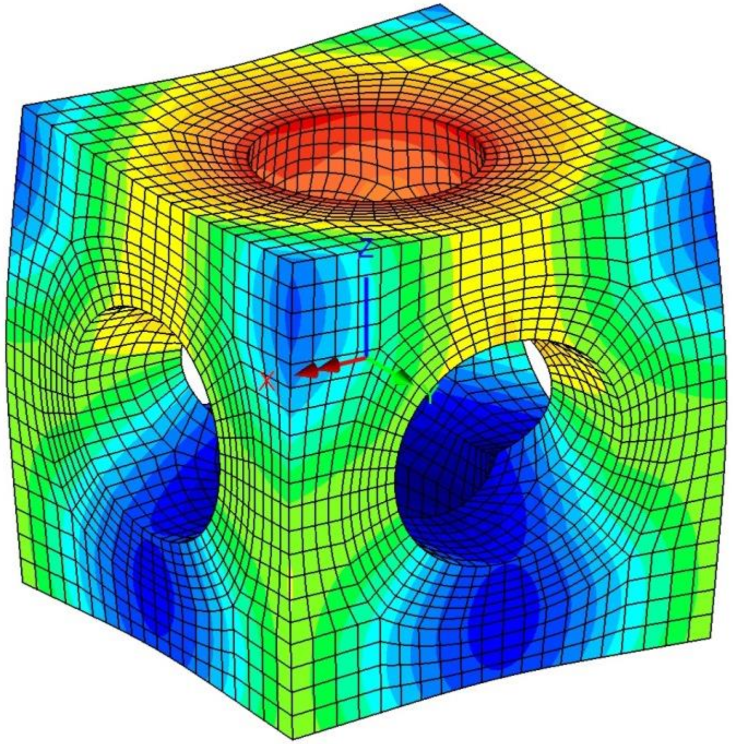




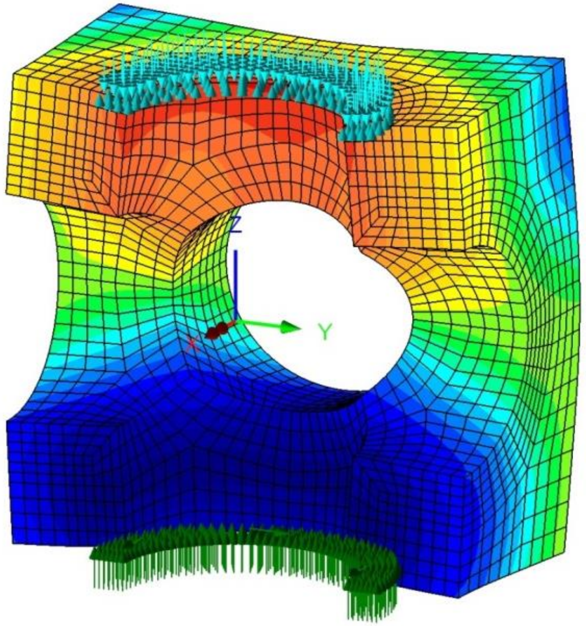


The bottom-half flow sheets and W14.1-2 are assembled, then W13 is added

What to use it for: Examples, A Structured Mesh

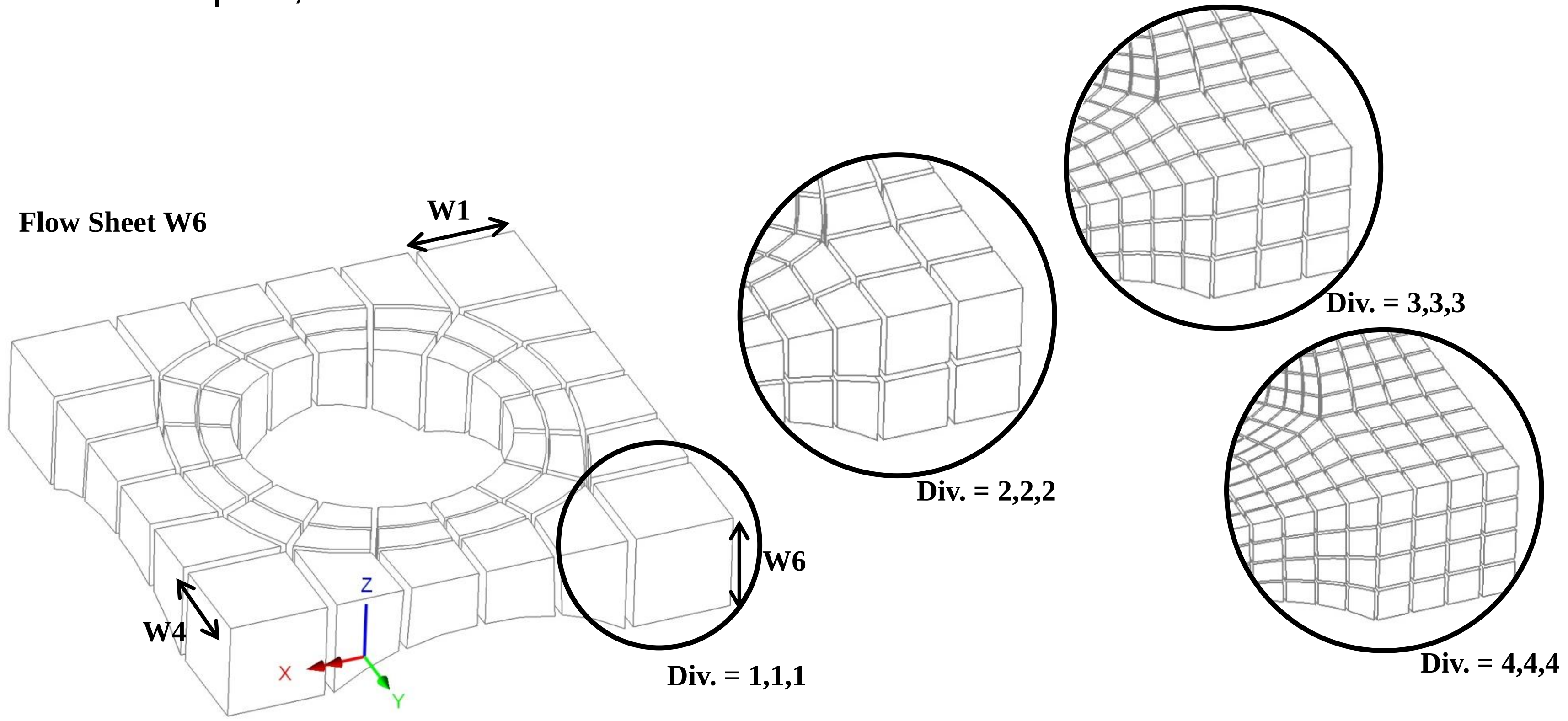


The complete set of flow sheets



Half the mesh cutting through a number of flow sheets …

What to use it for: Examples, Mesh Refinement

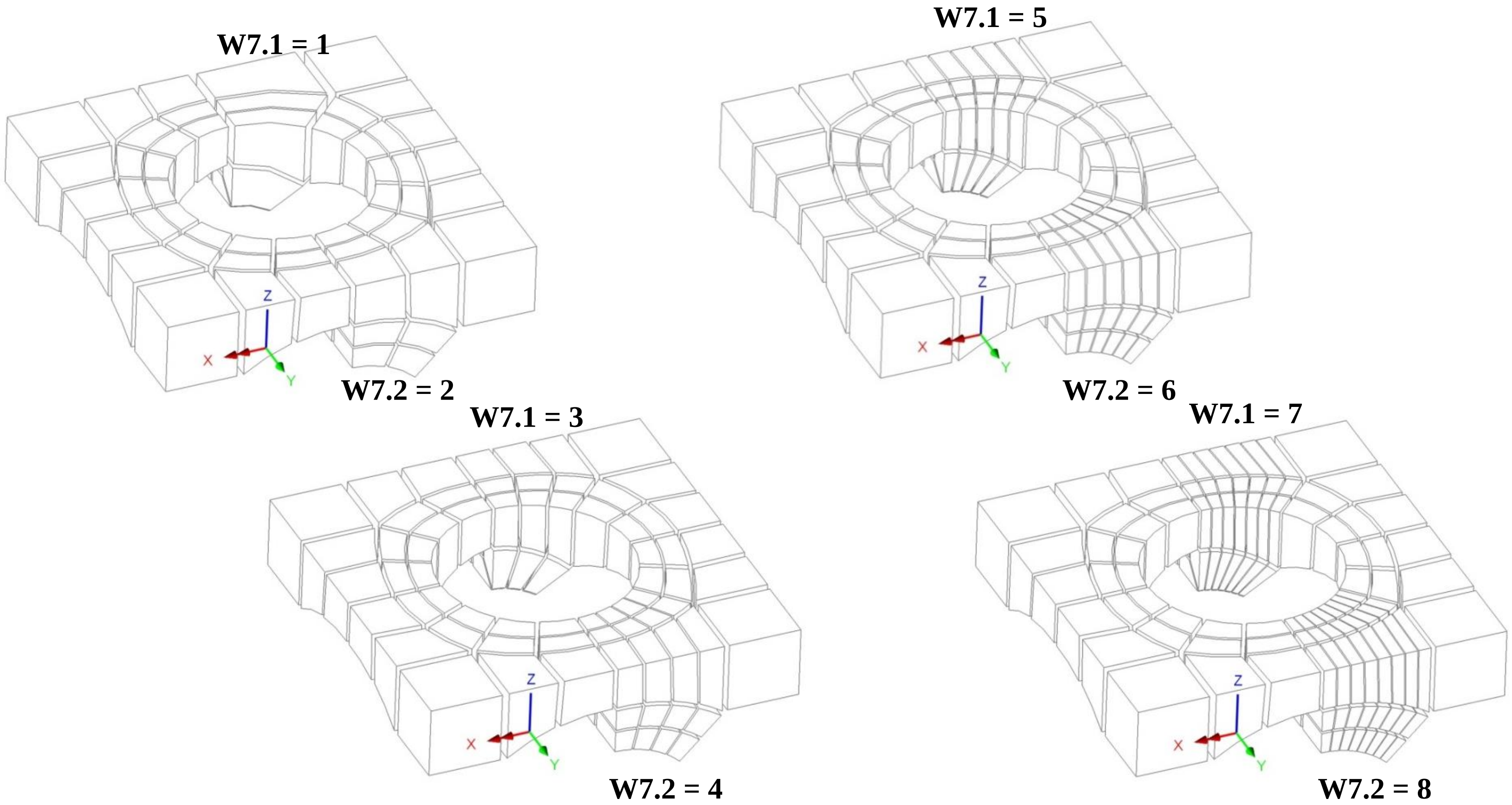


The Coarse Hexahedral Cluster has originally a set of flow sheet widths of “1” across all 37 of them, giving for example a set of division numbers for the hexahedron in the intersection between flow sheets W1, W4 and W6 (in the circle) of (1,1,1).

Uniform changes to the flow sheets widths for W1, W4 and W6 results in the division numbers shown in the subsequent circles: (2,2,2), (3,3,3) and (4,4,4). Meshes are created using traditional hex-in-hex regular meshing algorithms (h-convergence).

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

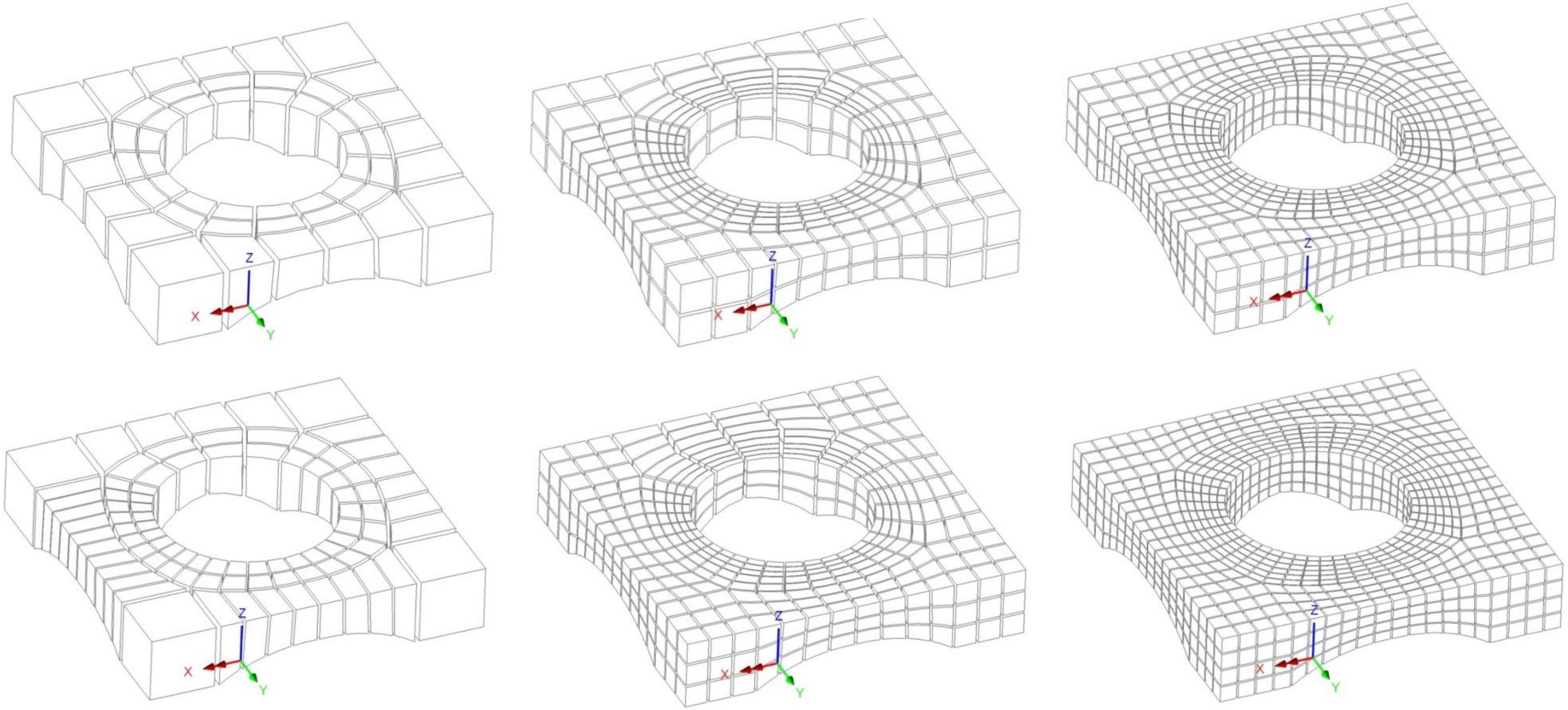
What to use it for: Examples , Mesh Refinement



Each of the 37 flow sheets are independent of any of the others and a change to any one of the widths is a delta change, i.e., most of the existing mesh can remain unchanged and only a small part needs re-computing, saving significant amount of man and computer time.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

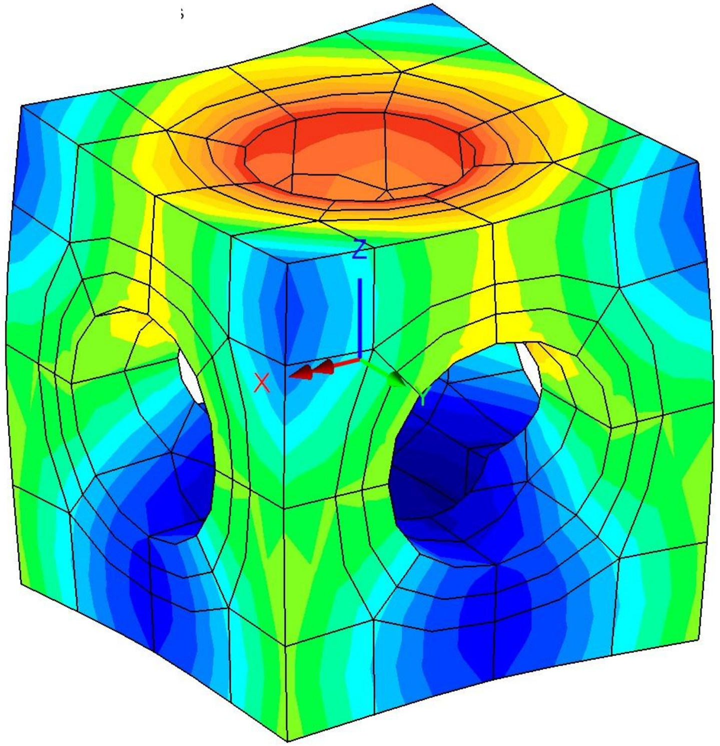
What to use it for: Examples, Mesh Refinement

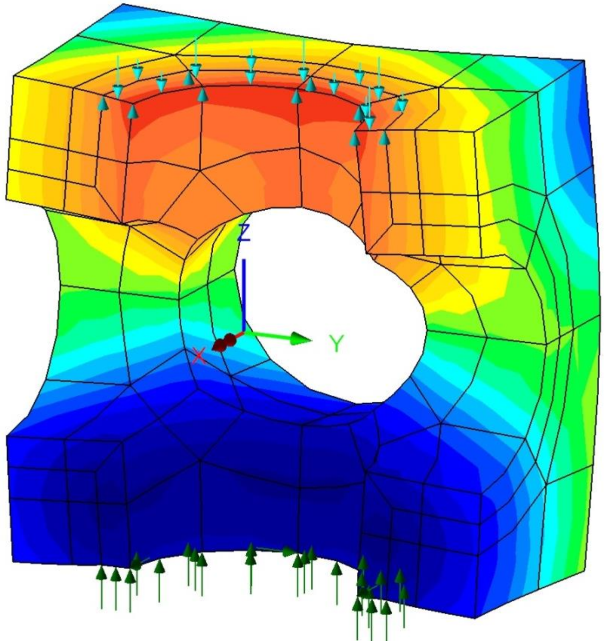


The initial Coarse Hexahedral Cluster provides a way to fill the volume with well-formed mesh flows and well-formed elements controlled by the presence (0 or 1) and width of the irregular edge networks. Any of the flow sheet widths can be varied in a systematic way to capture the deformation or stress gradients from the load distribution and/or particular boundary conditions or the a priori stress field prior to the first analysis.

The gradients in the analysis results in one analysis can be used to populate the Volume Mesh Controls in subsequent analyses. The feedback from previous analyses creates adaptive meshing, and forms part of a Convergence Study to find where the results no longer change due to mesh refinement.

What to use it for: Examples, Convergence Studies

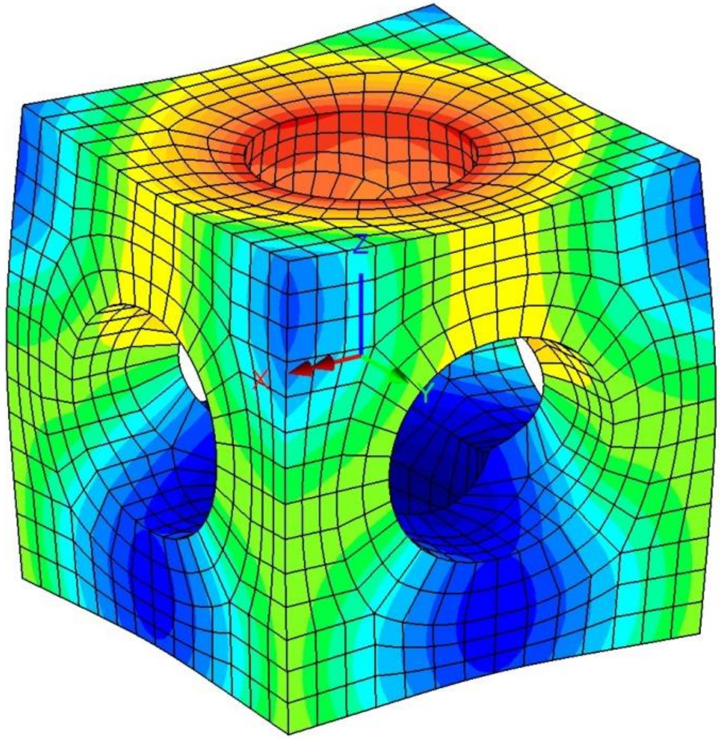


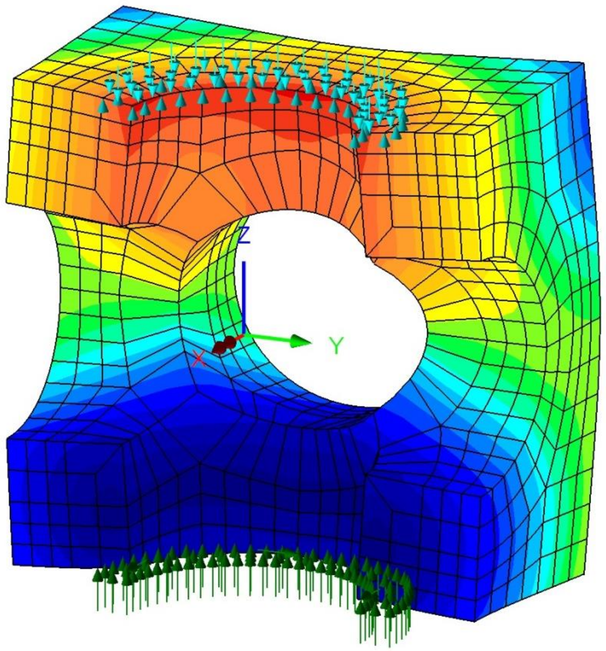


Model-01 Coarse Mesh:
Dx = Dy = Dz = 50.0
Holes radius: Dr = 25.00 mm
Off centre position: 0.0 mm

First analysis with the Coarse Hexahedral Cluster :
Model-01 Coarse Mesh: the original coarse mesh
(This may give warnings due to elements shape quality violations)

What to use it for: Examples, Convergence Studies

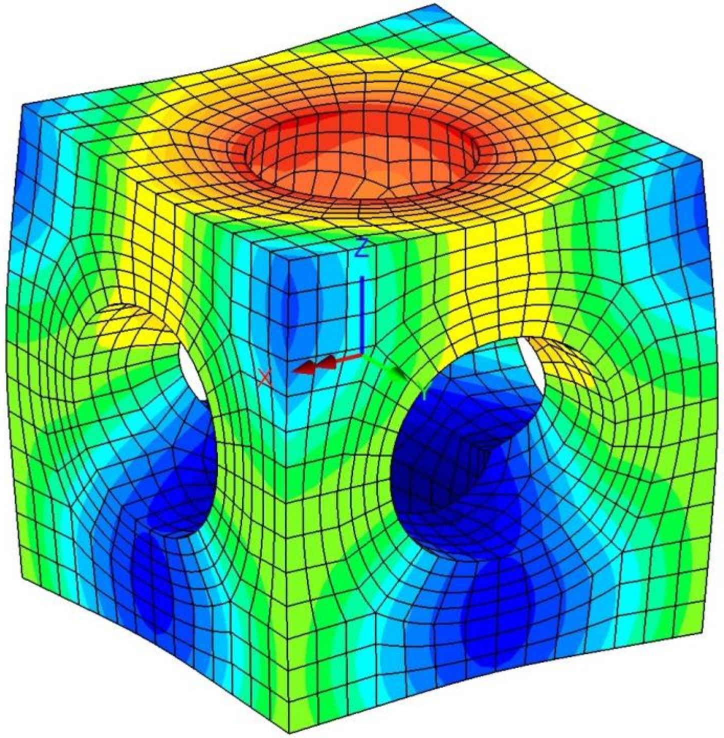


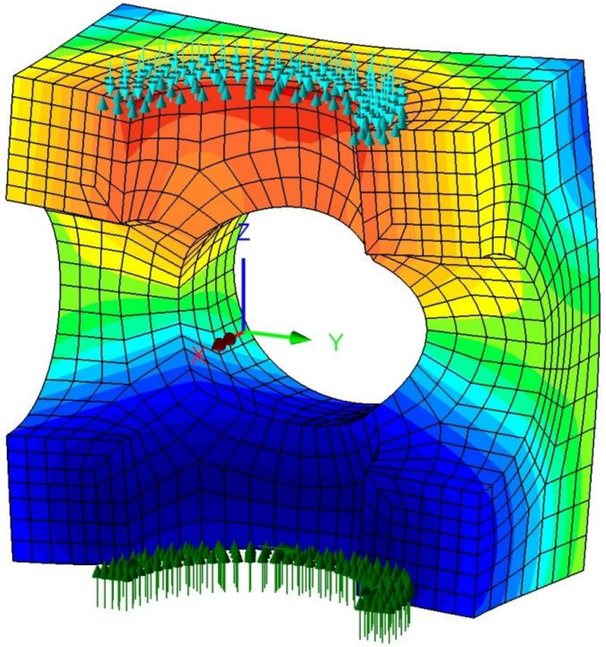


Model-01 Div 3, coarse centre:
Dx = Dy = Dz = 50.0
Holes radius: Dr = 25.00 mm
Off centre position: 0.0 mm

Second analysis with a finer mesh:
Model-01 Div 3: h-refinement, step 2
Results displayed on the finer mesh used in the analysis

What to use it for: Examples, Convergence Studies



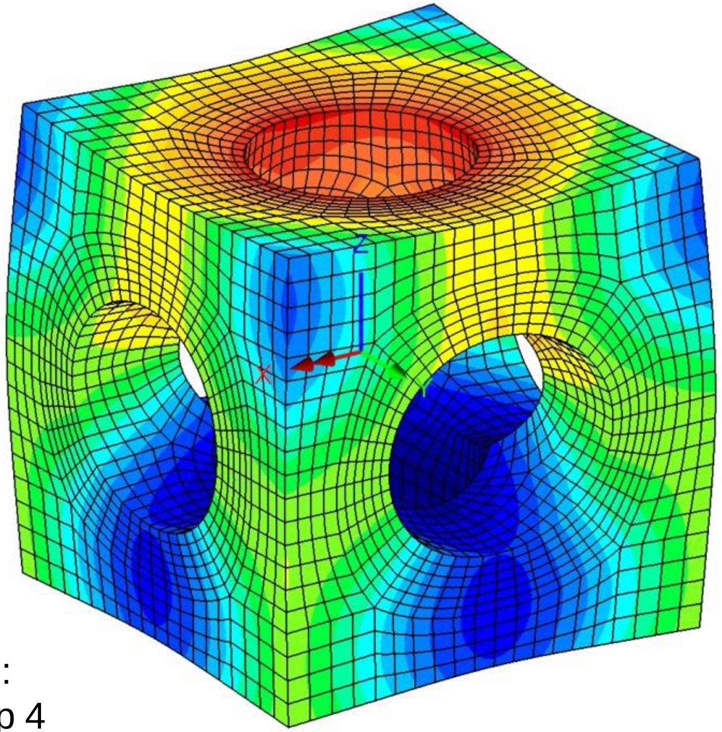


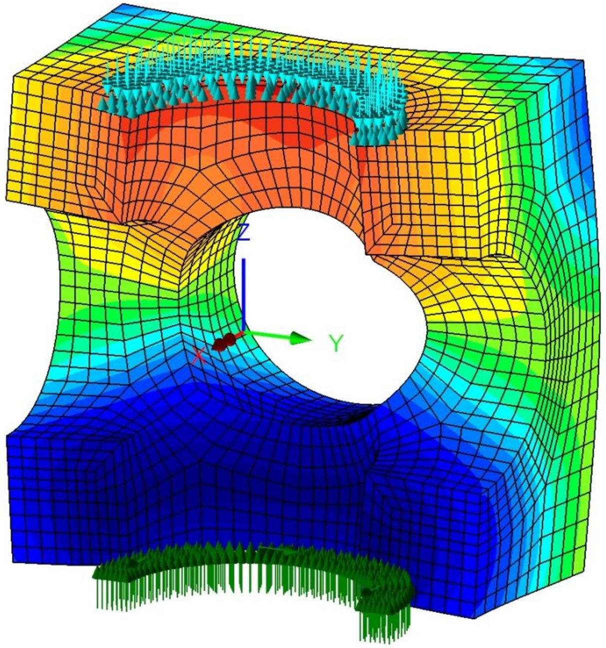
Model-01 Div 3, fine centre:
Dx = Dy = Dz = 50.0
Holes radius: Dr = 25.00 mm
Off centre position: 0.0 mm

Third analysis with a finer mesh:
Model-01 Core 2: h-refinement, step 3
Results displayed on the finer mesh used in the analysis

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Convergence Studies





Forth analysis with a yet finer mesh:
Model-01 Core 3: h-refinement, step 4
Results displayed on the finer mesh used in the analysis

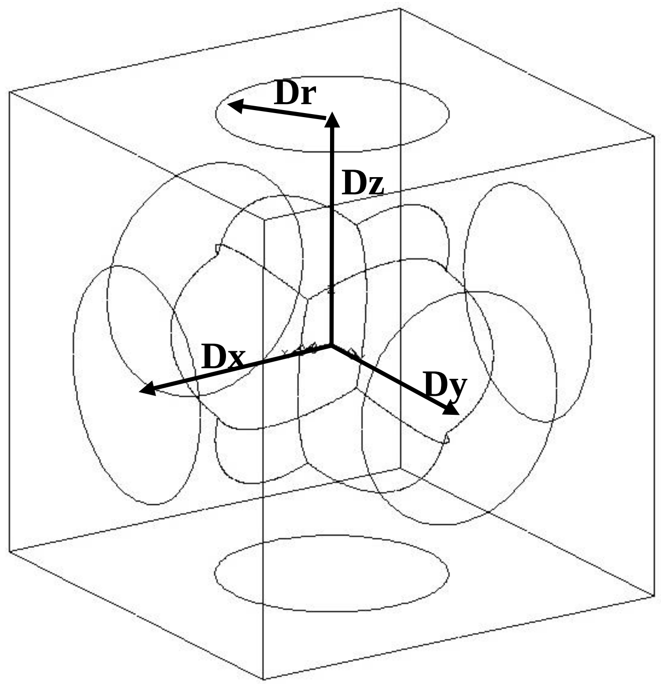
Model-01 Div 4, fine centre:
Dx = Dy = Dz = 50.0
Holes radius: Dr = 25.00 mm
Off centre position: 0.0 mm

Convergence can be automatically computed running a script that updates the flow sheet widths between each analysis, maintaining the structure of the mesh, i.e., the Coarse Hexahedral Cluster

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies

A parameterised version of the original shape:



With these parameters, it will be possible to compute the performance of a whole class of shapes with minimum of changes to the mesh specification:

The following information is reused for each set of parameters:
- The equation system for the assembly of eight 3D Kernels
- The irregular edge networks
- The load, boundary conditions, material properties.

The following is changed for each case:
- The coordinates for the points that are repositioned
- The flow sheet widths that need changing to maintain a mesh of well-formed elements

The analyses can be run in a sequence driven by a Script that for each step:
- Makes the necessary changes
- Creates the Analysis input file
- Runs the analysis
- Created the pictures and print them to file
- Store the Results-of-Interest in a RDBMS for the user to compare and plot performance graphs/surfaces across the analyses

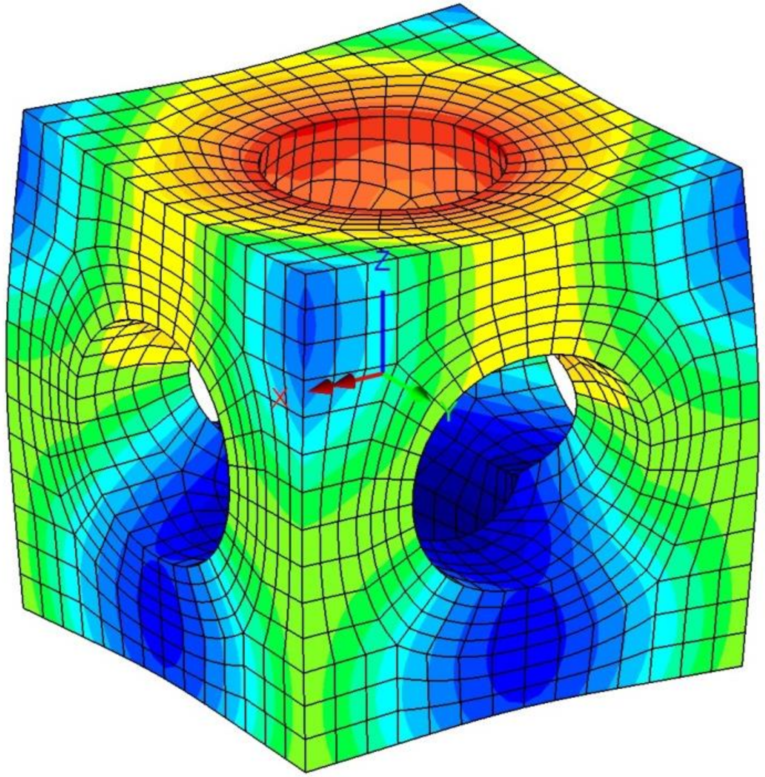
All “before lunch”.

The parameters controlling the positions of the holes:
- Dx distance of face from origin in x-direction
- Dy distance of face from origin in y-direction
- Dz distance of face from origin in z-direction

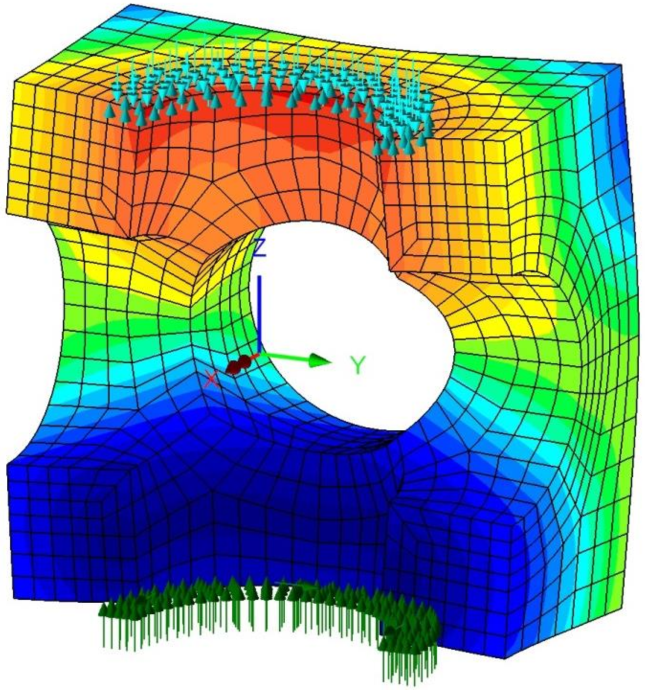
The parameters controlling the dimensions of the holes:
- Dr hole radius

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies



Model-01 Coarse Mesh: h-refinement, step 2
The position of the inner boundary (the holes) can be varied through a parameterised study of the effect of change in the position of the intersection point between the cylindrical holes.

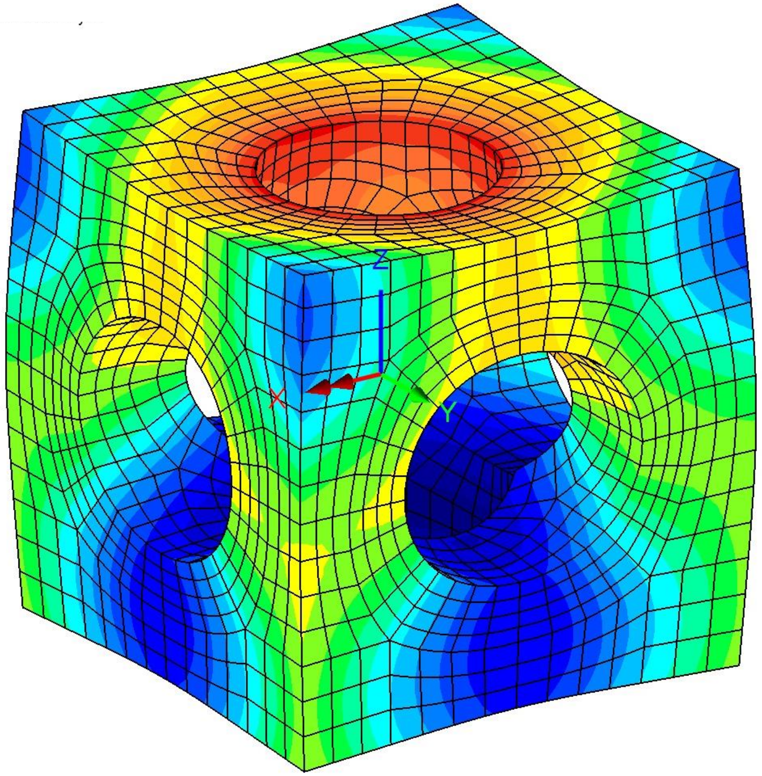


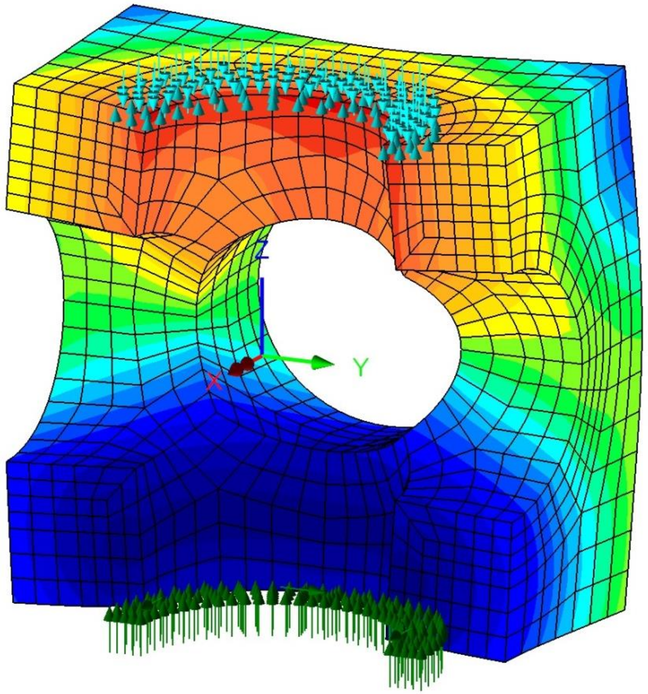
Model-01 Core 2: XYZ 00:
Dx = Dy = Dz = 50.0 mm
Holes radius: Dr = 25.00 mm
Off centre position: 0.0 mm

All the plots use a global set of values for the contours

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies





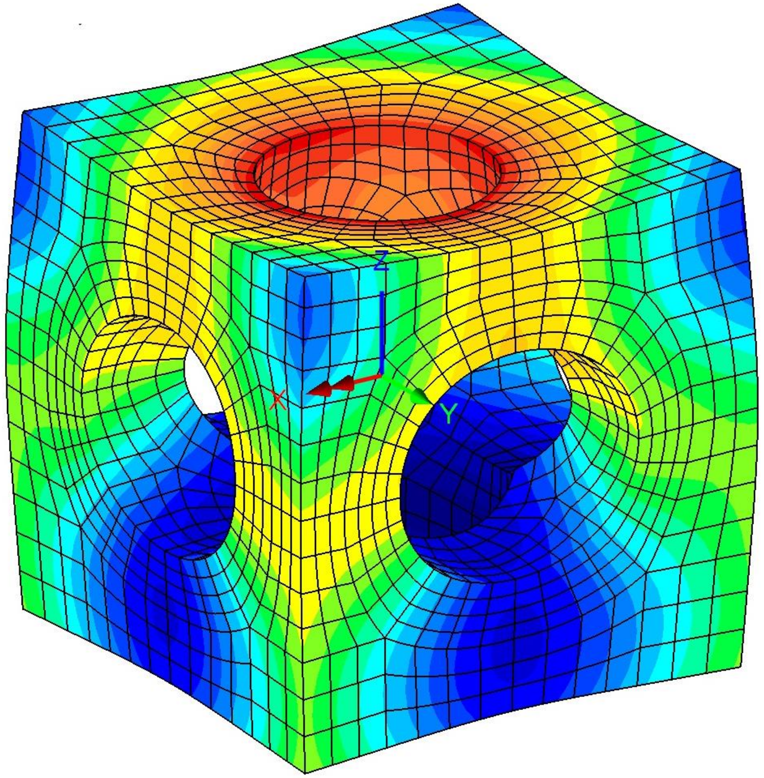
Model-01 Core 2: XYZ 01:
Dx = Dy = Dz = 48.75 mm
Holes radius: Dr = 25.00 mm
Off centre position: 1.25 mm

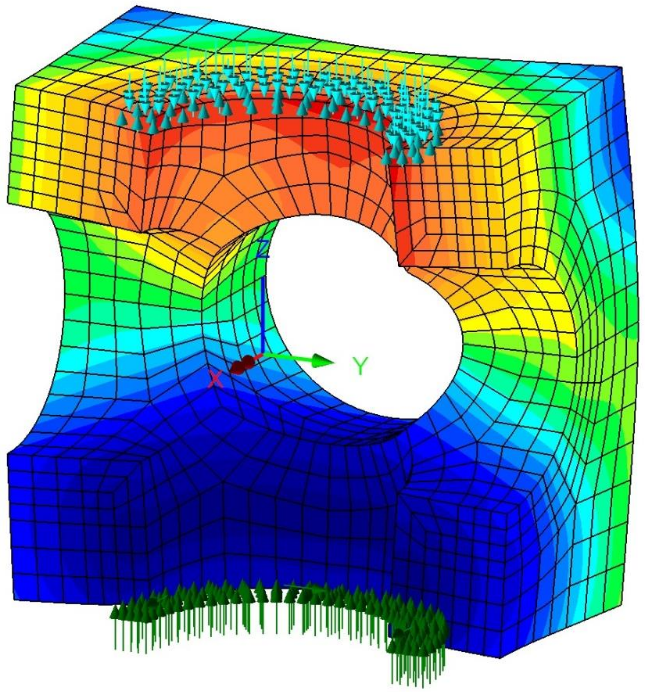
The shape has changed without changing the topology, so the position of the irregular edge networks and flow sheets are all the changes that affect the shape.
The equation system remains unchanged.
All widths are kept the same throughout.

All the plots use a global set of values for the contours

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies





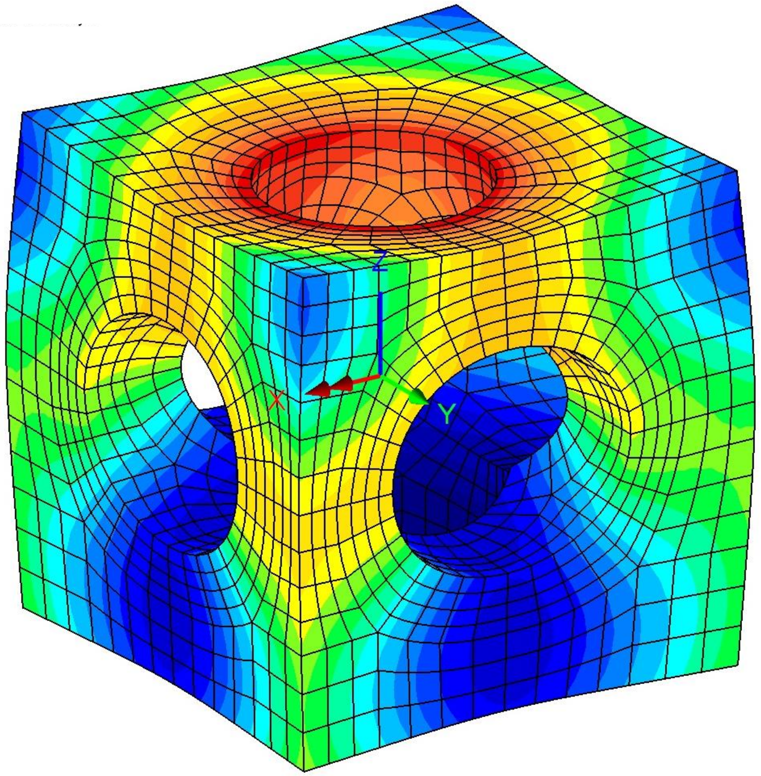
Model-01 Core 2: XYZ 02:
Dx = Dy = Dz = 47.5 mm
Holes radius: Dr = 25.00 mm
Off centre position: 2.50 mm

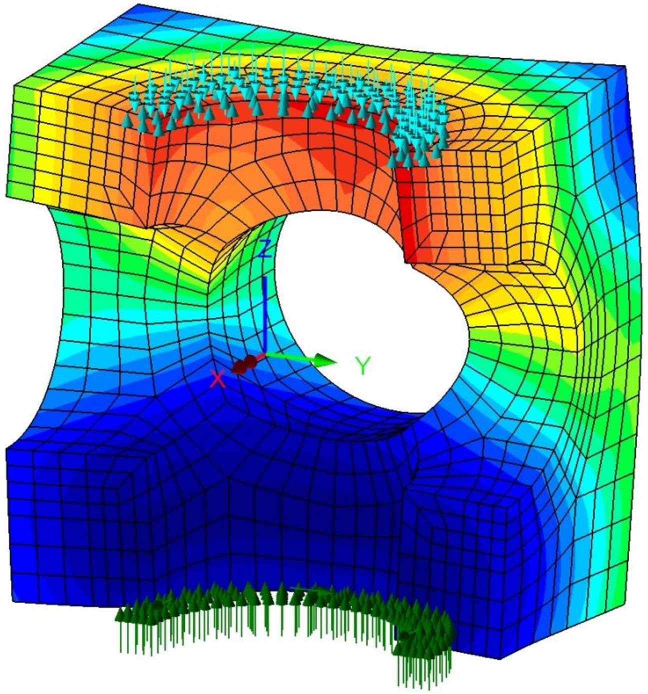
The set of irregular edge networks is the same, so the same solution can be used, only changing the widths of the flow sheets for better positioning of the irregular edge networks.

All the plots use a global set of values for the contours

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies





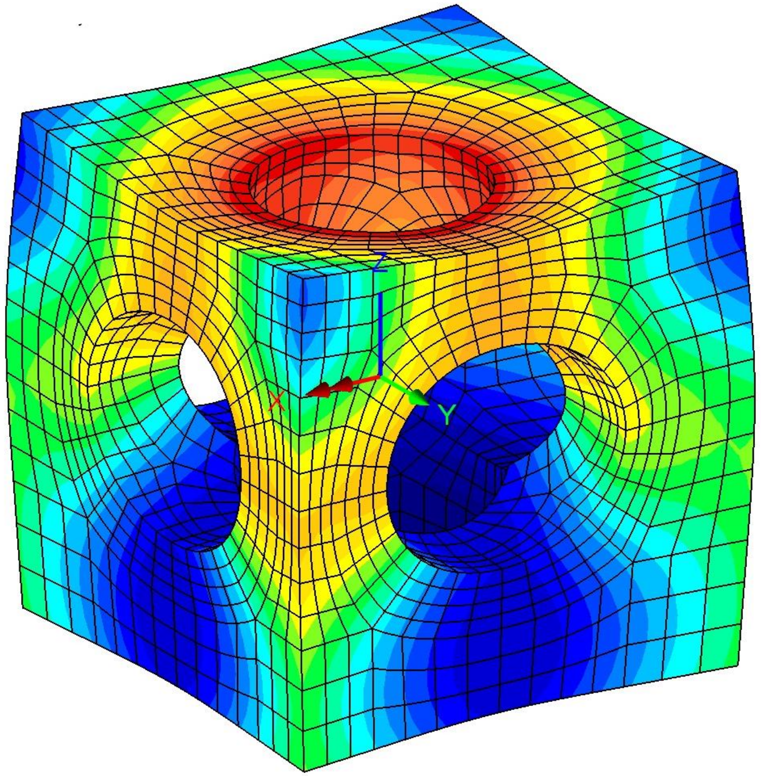
Model-01 Core 2: XYZ 03:
Dx = Dy = Dz = 46.25 mm
Holes radius: Dr = 25.00 mm
Off centre position: 3.75 mm

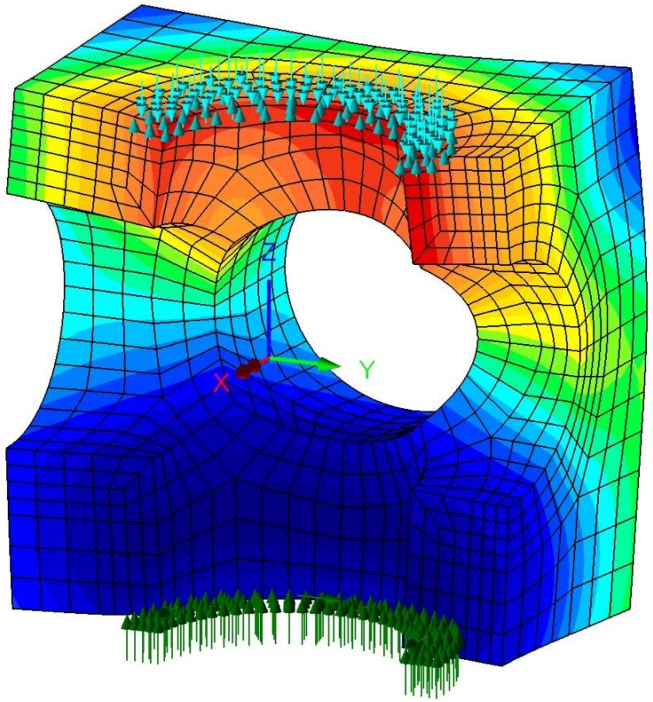
As can be seen, the flow sheet widths can remain unchanged between the parametric steps, making the delta change only a matter of repositioning the nodes in the mesh.

All the plots use a global set of values for the contours

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies





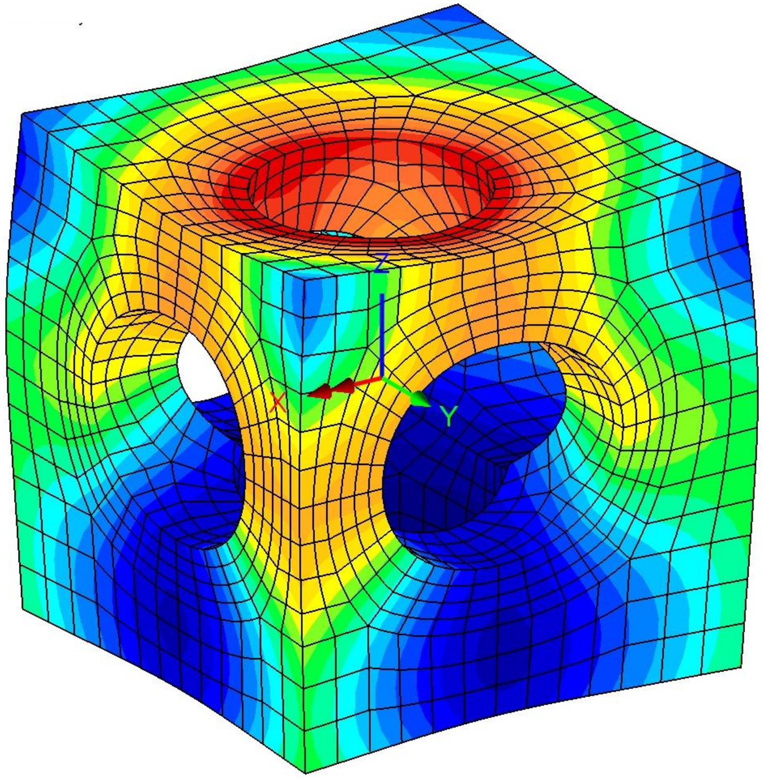
Model-01 Core 2: XYZ 04:
Dx = Dy = Dz = 45.0 mm
Holes radius: Dr = 25.00 mm
Off centre position: 5.00 mm

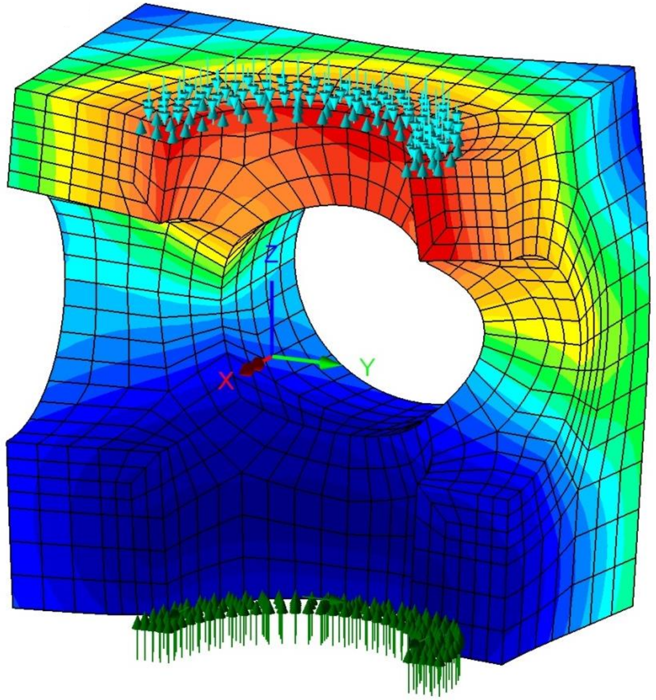
A check on element shape quality will eventually require a change to the flow sheet widths.

All the plots use a global set of values for the contours

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

What to use it for: Examples, Parameter Studies



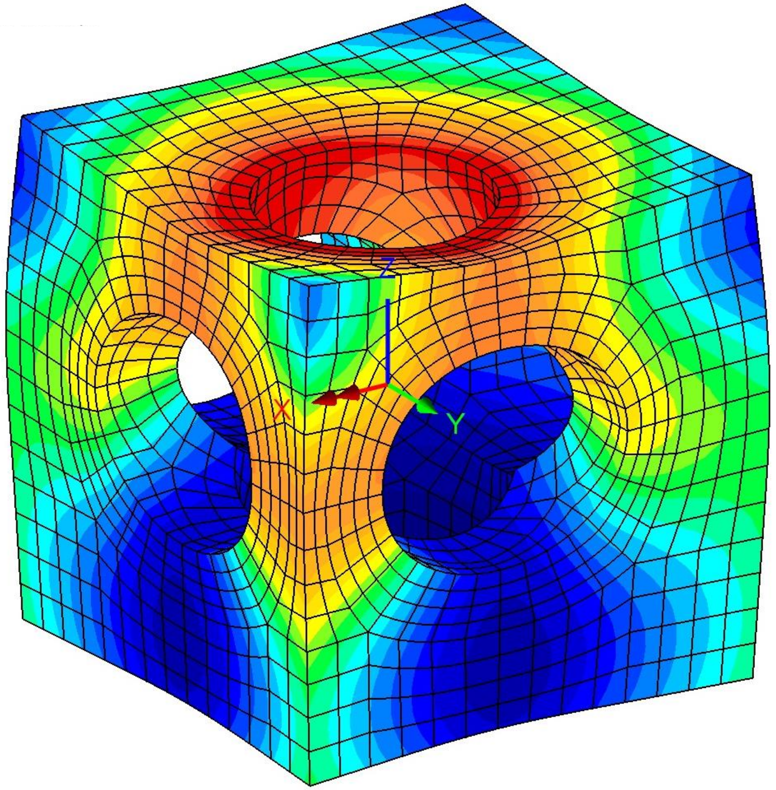


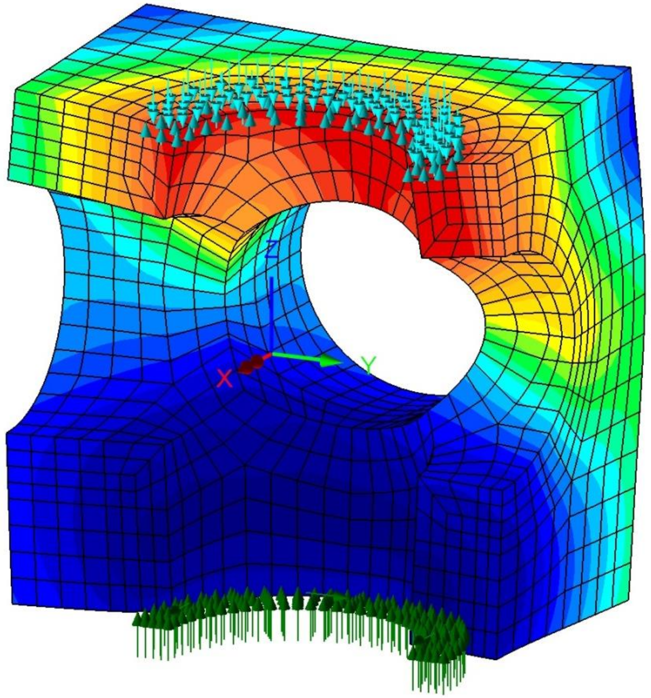
Model-01 Core 2: XYZ 05:
Dx = Dy = Dz = 43.75 mm
Holes radius: Dr = 25.00 mm
Off centre position: 6.25 mm

For larger changes, it may be beneficiary to introduce additional internal flow sheets to separate the irregular edge networks further, to maintain high element shape quality.

All the plots use a global set of values for the contours

What to use it for: Examples, Parameter Studies



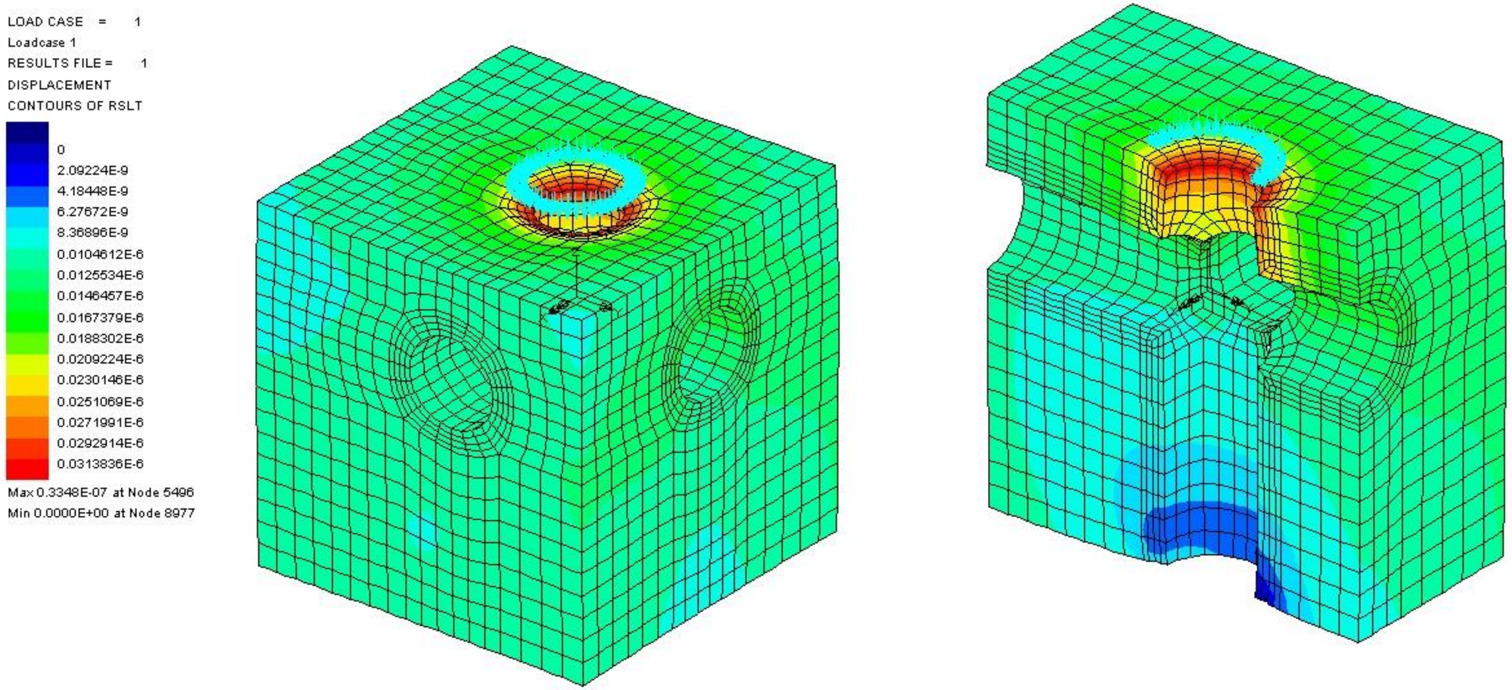


Model-01 Core 2: XYZ 06:
Dx = Dy = Dz = 42.5 mm
Holes radius: Dr = 25.00 mm
Off centre position: 7.50 mm

Separation of the irregular edge networks is not demonstrated here.

All the plots use a global set of values for the contours

What to use it for: Examples, Parameter Studies



Other parameter studies may include
- varying the Dz separately and
- varying the hole diameters

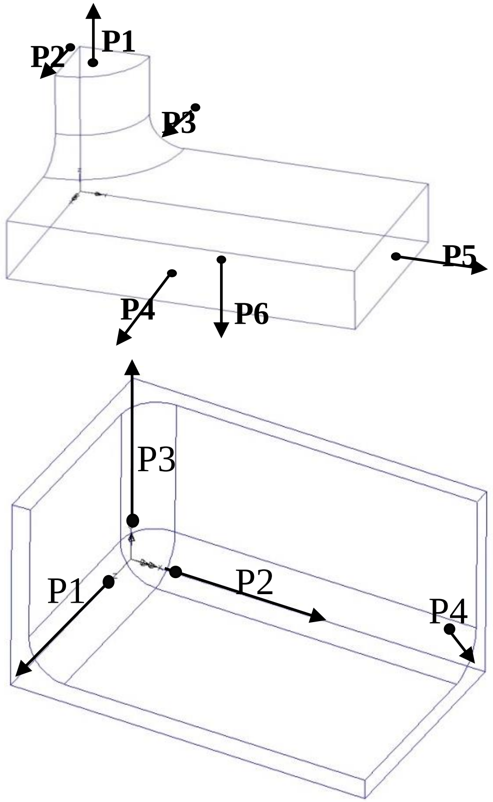
The cluster of 3D Kernels can be represented in one underdetermined super set equation system that can be used for a large number of variations:

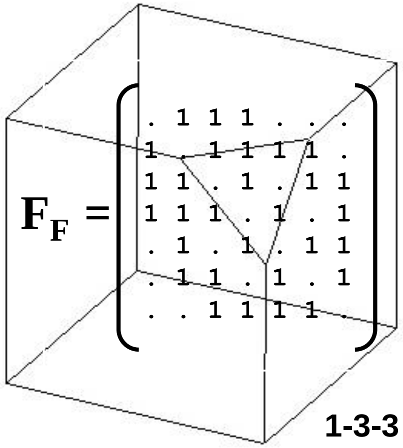
- The irregular edge networks used in a shape, their positions and separations
- The width of the flow sheets between the irregular edge networks
- Convergence studies;
- Parameter studies varying geometry features, as in this case the position of the intersection between three cylinders, and the diameters of the holes;

Separation of the irregular edge networks is not demonstrated here.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

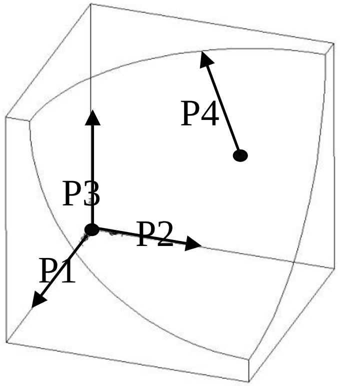
What to use it for: Examples, Shared Topology





$$F_F = \begin{pmatrix} . & 1 & 1 & 1 & . & . & . \\ 1 & . & 1 & 1 & 1 & 1 & . \\ 1 & 1 & . & 1 & . & 1 & 1 \\ 1 & 1 & 1 & . & 1 & . & 1 \\ . & 1 & . & 1 & . & 1 & 1 \\ . & 1 & 1 & . & 1 & . & 1 \\ . & . & 1 & 1 & 1 & 1 & . \end{pmatrix}$$

1-3-3



$$\begin{matrix} \left(A_e \mid A_i \right) \\ \left(G \right) \end{matrix} \begin{pmatrix} w_e \\ w_i \end{pmatrix} - \left(I \right) \left(L \right) = 0$$

Shared topology across various shapes and across all their parameter sets results in a shared equation system

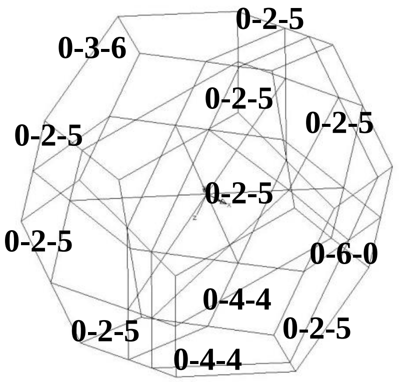
Dislocation Meshing is based on the axiom of topology first, geometry second. Topology, i.e., how the element mesh flows within the cluster of cells, is the first thing to sort out. Geometry, i.e., calculating the positions of vertices, edges and faces when embedded in points, curves and surfaces is secondary.

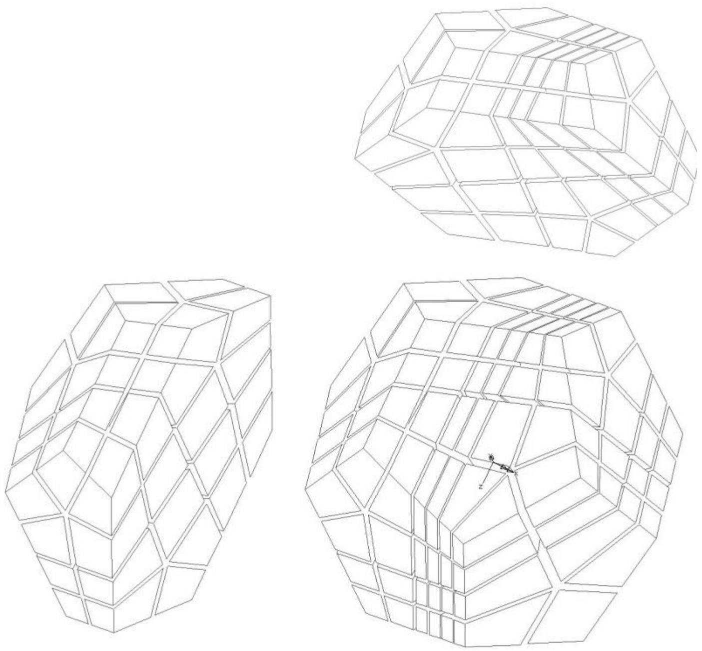
Hence, the same F_F matrix is shared between a number of shapes, also when faces are combined faces as above left.
The sequencing of vertices, edges and faces determines the sequencing of faces in F_F and then the order of the flow sheets in the equation system, but not the content of them. The structure of the equation system is invariant to numbering schemes.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

The General Theory of Representation and Manipulation of Shapes

Examples of tessellations to create a Coarse Hexahedral in 3D Kernels

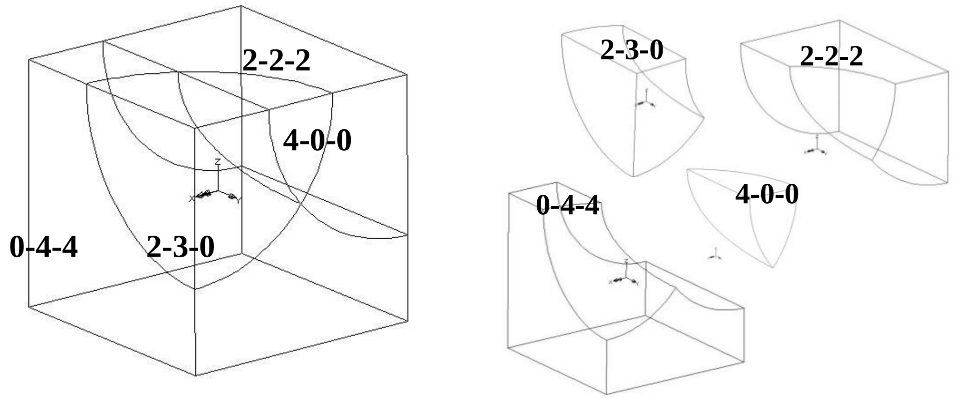


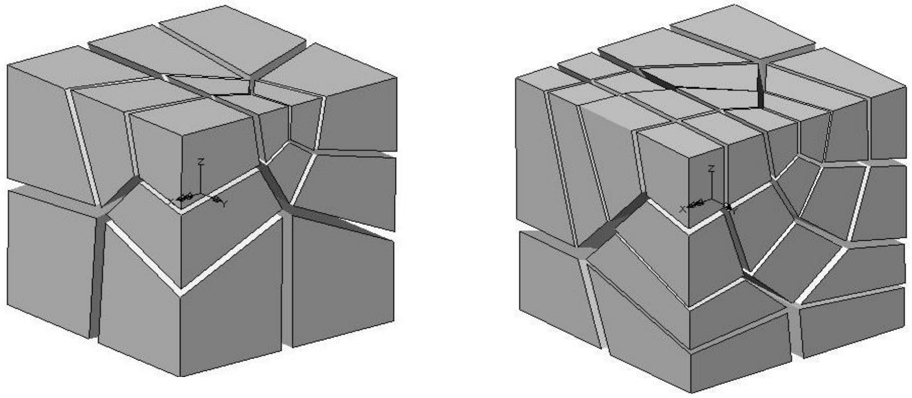


A Dodecahedron topology is tessellated into a Cellular Non-manifold Boundary-Representation Cell Cluster by using four internal flow sheets

Using MPSD as an Operator, a Coarse Hexahedral Cluster is created consisting of Hexahedra only

Tessellation can start from more complex shapes and end up with a
Cluster of 3D Kernels: any combination of 3D Kernels, Prisms only, and/or Hexahedra only

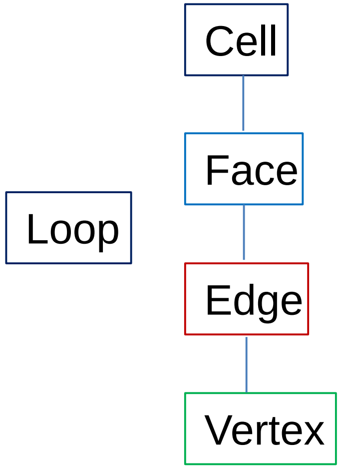




A Cube can be cut by two intersecting surfaces:
- A sphere and a cylinder (one particular case shown here)
- creating a cluster of 3D Kernels

Two Coarse Hexahedral Clusters are shown, the right hand one is the MPSD

The General Theory of Representation and Manipulation of Shapes



	V	E	F	C
V	x	x	x	x
E		x	x	x
F			x	x
C				x

The hierarchy of topological entities

The adjacency matrices for a b-rep solid model:
V_v – the vertices that are adjacent to other vertices
V_E – the vertices at the “corners” of an edge
V_F – the vertices at the “corners” of a face
V_C – the vertices at the “corners” of a cell
E_E – the edges that are adjacent to other edges
E_F – the edges that are adjacent to faces
E_C – the edges that are adjacent to cells
F_F – the faces that are adjacent to other faces
F_C – the faces that are adjacent to cells
C_C – the cells that are adjacent to other cells

The adjacency relationships thus created

The relationships between the adjacency matrices

$F_D = F_V * V_E$ where $F_D = F_E^A + 2*F_E^O$ where

F_E^A a Face-Edge Adjacent matrix
$2*F_E^O$ a Face-Edge Opposite matrix times two

And by extracting the F_E^A matrix:

$F_F + D = F_E^A * (F_E^A)^T$ where D – a diagonal matrix with the diagonal values equal the number of edges in the respective face

or

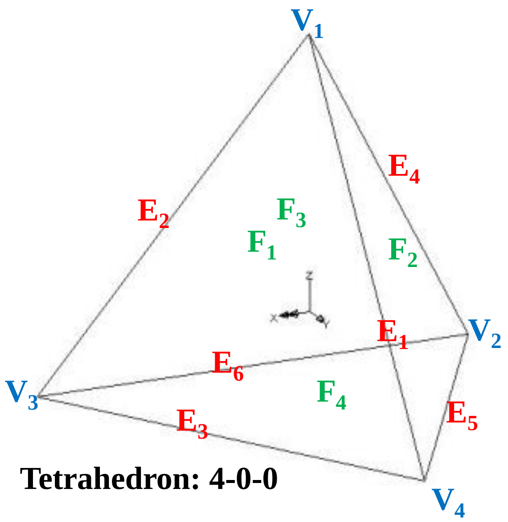
$F_F = F_E^A * (F_E^A)^T - D$

$(F_E^A)^T$ – the Face-Edge adjacency matrix transposed

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



Tetrahedron: 4-0-0

$$F_F = \begin{bmatrix} . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \\ 1 & 1 & 1 & . \end{bmatrix} \quad E_F = \begin{bmatrix} 1 & 1 & . & . \\ 1 & . & 1 & . \\ 1 & . & . & 1 \\ . & 1 & 1 & . \\ . & 1 & . & 1 \\ . & . & 1 & 1 \end{bmatrix} \quad V_E = \begin{bmatrix} 1 & 1 & . & 1 & . & . \\ . & . & . & 1 & 1 & 1 \\ . & 1 & 1 & . & . & 1 \\ 1 & . & 1 & . & 1 & . \end{bmatrix} \quad F_V = \begin{bmatrix} 1 & 1 & 1 & . \\ . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \end{bmatrix}$$

$$E_F = F_E^T$$

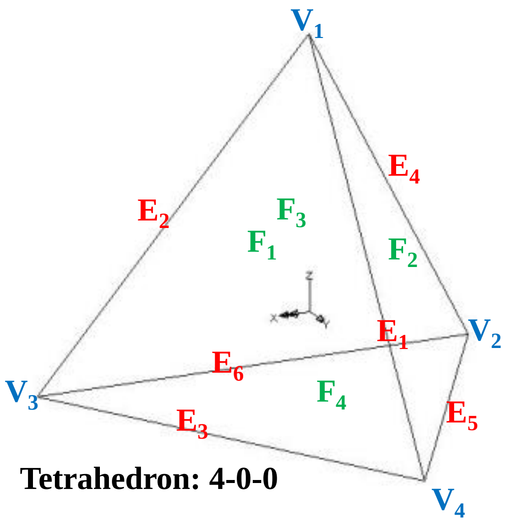
Four of the relationship matrices for a 4-0-0:

These matrices can be extracted from the F_F matrix or by interrogating the data stored for the shape.

An example: Four of the relationship matrices for a Tetrahedron called a 4-0-0. The adjacency matrices defined so far relate to each other in an algorithmic way. The following relationships exist:

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



Tetrahedron: 4-0-0

$$F_V = \begin{bmatrix} 1 & 1 & 1 & . \\ . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \end{bmatrix} \quad V_E = \begin{bmatrix} 1 & 1 & . & 1 & . & . \\ . & . & . & 1 & 1 & 1 \\ . & 1 & 1 & . & . & 1 \\ 1 & . & 1 & . & 1 & . \end{bmatrix} \quad F_D = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 1 & 2 & 2 \\ 2 & 2 & 2 & 1 & 1 & 1 \\ 2 & 1 & 1 & 2 & 2 & 1 \end{bmatrix}$$

$$F_F = \begin{bmatrix} . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \\ 1 & 1 & 1 & . \end{bmatrix} \quad F_F + D = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \quad F_E{}^A = \begin{bmatrix} 1 & . & 1 & . & 1 & . \\ 1 & 1 & . & 1 & . & . \\ . & . & . & 1 & 1 & 1 \\ . & 1 & 1 & . & . & 1 \end{bmatrix} \quad F_E{}^O = \begin{bmatrix} . & 1 & . & 1 & . & 1 \\ . & . & 1 & . & 1 & 1 \\ 1 & 1 & 1 & . & . & . \\ 1 & . & . & 1 & 1 & . \end{bmatrix}$$

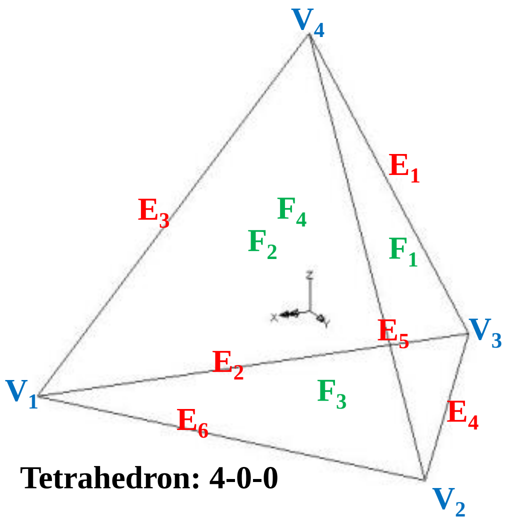
An example: Four of the relationship matrices for a Tetrahedron called a 4-0-0. The F_F adjacency matrix can be found from other adjacency matrix relationships for the same topology, the F_V and the V_E matrices.

The order in which the faces, edges and vertices are represented is of no significance. For the representation of the F_F matrix, the order in which the faces appear will only create a different appearance of the matrix, a different perturbation, without changing the mathematical correctness and content of the information represented.

Any perturbation will dependent on the actual implementation in a computer system and the result will not be influenced by the numbering schemes used for any of the entity types. The computations based on the matrices are the same irrespective of the perturbations used for any of the actual entities, i. e. , the result is invariant to the implementation environment.

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



Tetrahedron: 4-0-0

$$F_V = \begin{bmatrix} . & 1 & 1 & 1 \\ 1 & 1 & . & 1 \\ 1 & 1 & 1 & . \\ 1 & . & 1 & 1 \end{bmatrix} \quad V_E = \begin{bmatrix} . & 1 & 1 & . & . & 1 \\ . & . & . & 1 & 1 & 1 \\ 1 & 1 & . & 1 & . & . \\ 1 & . & 1 & . & 1 & . \end{bmatrix} \Rightarrow F_D = \begin{bmatrix} 2 & 1 & 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 1 & 2 & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 & 1 & 1 \end{bmatrix}$$

$$\Downarrow$$

$$F_E^{A} = \begin{bmatrix} . & 1 & 1 & . & . & 1 \\ 1 & 1 & . & 1 & . & . \\ 1 & . & 1 & . & 1 & . \\ . & . & . & 1 & 1 & 1 \end{bmatrix} \quad F_E^{O} = \begin{bmatrix} 1 & . & . & 1 & 1 & . \\ . & . & 1 & . & 1 & 1 \\ . & 1 & . & 1 & . & 1 \\ 1 & 1 & 1 & . & . & . \end{bmatrix}$$

$$\Leftarrow$$

$$F_F = \begin{bmatrix} . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \\ 1 & 1 & 1 & . \end{bmatrix} \quad F_F + D = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$

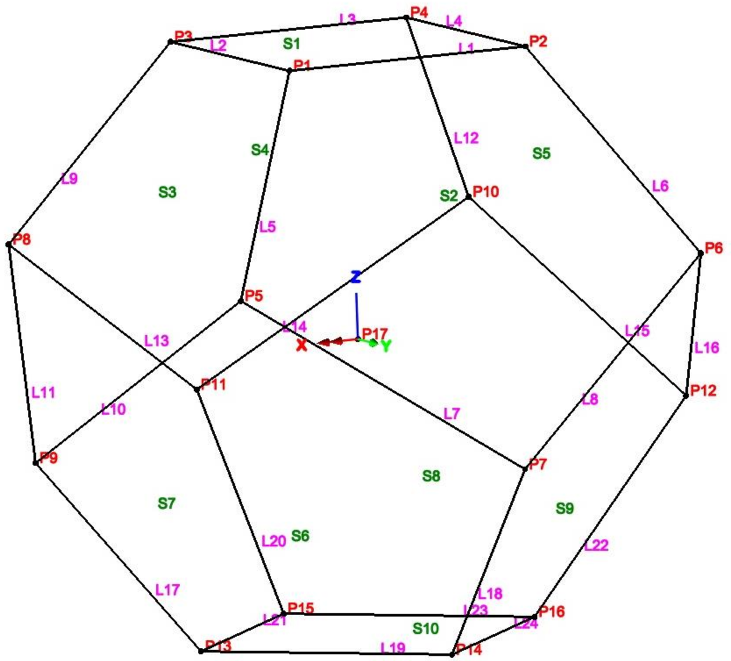
An example: Alternative sequencing: The F_F adjacency matrix can be found from other adjacency matrix relationships for the same topology, the F_V and the V_E matrices.

Alternative sequencing schemes used in the V, E and F matrices will give a valid result for the F_F matrix.
The fact that the F_F matrices for a tetrahedron (4-0-0) are the same using two different sequencing schemes stems from the fact that each face has to be the neighbour to all the other faces, 4-0-0 is a special case.
For other cells with more faces, the F_F matrix will be different for different numbering schemes.

The F_F matrix can be the starting point in a calculation of the adjacency relationships in other matrices, as their content can be extracted from the original compact F_F format.

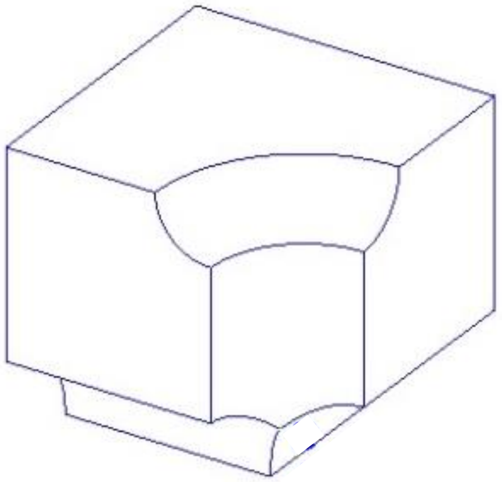
The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



The 3D Kernel 0-2-8

$$F_V = \begin{bmatrix}
1 & 1 & 1 & 1 & . & . & . & . & . & . & . & . & . & . & . & . \\
1 & 1 & . & . & 1 & 1 & 1 & . & . & . & . & . & . & . & . & . \\
1 & . & 1 & . & 1 & . & . & 1 & 1 & . & . & . & . & . & . & . \\
. & . & 1 & 1 & . & . & . & 1 & . & 1 & 1 & . & . & . & . & . \\
. & 1 & . & 1 & . & 1 & . & . & . & . & 1 & . & 1 & . & . & . \\
. & . & . & . & 1 & . & 1 & . & 1 & . & . & . & 1 & 1 & . & . \\
. & . & . & . & . & . & . & 1 & 1 & . & 1 & . & 1 & . & 1 & . \\
. & . & . & . & . & . & . & . & . & 1 & 1 & 1 & . & . & 1 & 1 \\
. & . & . & . & . & 1 & 1 & . & . & . & . & 1 & . & 1 & . & 1 \\
. & . & . & . & . & . & . & . & . & . & . & . & 1 & 1 & 1 & 1
\end{bmatrix}$$



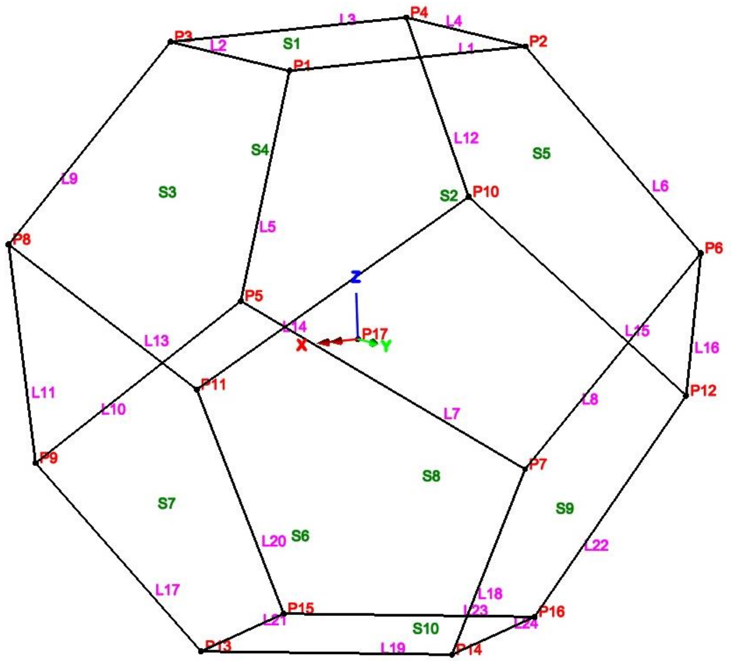
A geometry with the 0-2-8 as its topology

An example: The cellular model called a 0-2-8 has a particular computer implementation with the vertices, edges and faces as given in the picture above.
The matrices to create the F_F matrix explicitly can be based on the multiplication of the necessary F_V and V_E adjacency matrices, which can be extracted from the shape as stored in the computer program.

The F_V and the V_E matrices are shown in this and the following page, and the resulting F_F matrix is shown in the subsequent page.

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



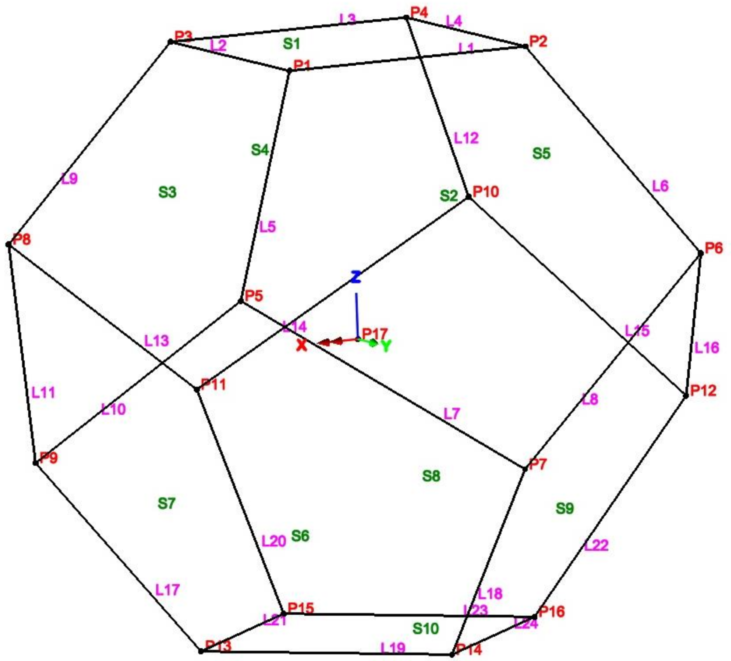
The 3D Kernel 0-2-8

$$V_E = \begin{bmatrix}
1&1&.&.&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.\\
1&.&.&1&.&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.\\
.&1&1&.&.&.&.&.&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.\\
.&.&1&1&.&.&.&.&.&.&.&1&.&.&.&.&.&.&.&.&.&.&.&.\\
.&.&.&.&1&.&1&.&.&1&.&.&.&.&.&.&.&.&.&.&.&.&.&.\\
.&.&.&.&.&1&.&1&.&.&.&.&.&.&.&1&.&.&.&.&.&.&.&.\\
.&.&.&.&.&.&1&1&.&.&.&.&.&.&.&.&.&1&.&.&.&.&.&.\\
.&.&.&.&.&.&.&.&1&.&1&.&1&.&.&.&.&.&.&.&.&.&.&.\\
.&.&.&.&.&.&.&.&.&1&1&.&.&.&.&.&1&.&.&.&.&.&.&.\\
.&.&.&.&.&.&.&.&.&.&.&1&.&1&1&.&.&.&.&.&.&.&.&.\\
.&.&.&.&.&.&.&.&.&.&.&.&1&1&.&.&.&.&.&1&.&.&.&.\\
.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&1&.&.&.&.&.&1&.&.\\
.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&.&1&.&1&.&.&.\\
.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&1&.&.&.&.&1\\
.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&1&.&1&.\\
.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&.&1&1&1
\end{bmatrix}$$

In general, the F_F and other adjacency matrices do not exist explicitly in a solid modelling environment. However, the vertex and edge relationships can be extracted and the F_F matrix can be created. The F_F matrix forms the basis for a mathematical theory for tessellation and meshing of solid models, as well as assembly and merging of cell clusters. The theory is called **Topology Operator Algebra**, while a search is on for similar research efforts.

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices



The 3D Kernel 0-2-8

$$F_F = \begin{bmatrix} . & 1 & 1 & 1 & 1 & . & . & . & . & . \\ 1 & . & 1 & . & 1 & 1 & . & . & 1 & . \\ 1 & 1 & . & 1 & . & 1 & 1 & . & . & . \\ 1 & . & 1 & . & 1 & . & 1 & 1 & . & . \\ 1 & 1 & . & 1 & . & . & . & 1 & 1 & . \\ . & 1 & 1 & 1 & . & . & 1 & . & 1 & 1 \\ . & . & 1 & . & . & 1 & . & 1 & . & 1 \\ . & . & . & 1 & 1 & . & 1 & . & 1 & 1 \\ . & 1 & . & . & 1 & 1 & . & 1 & . & 1 \\ . & . & . & . & . & 1 & 1 & 1 & 1 & . \end{bmatrix}$$

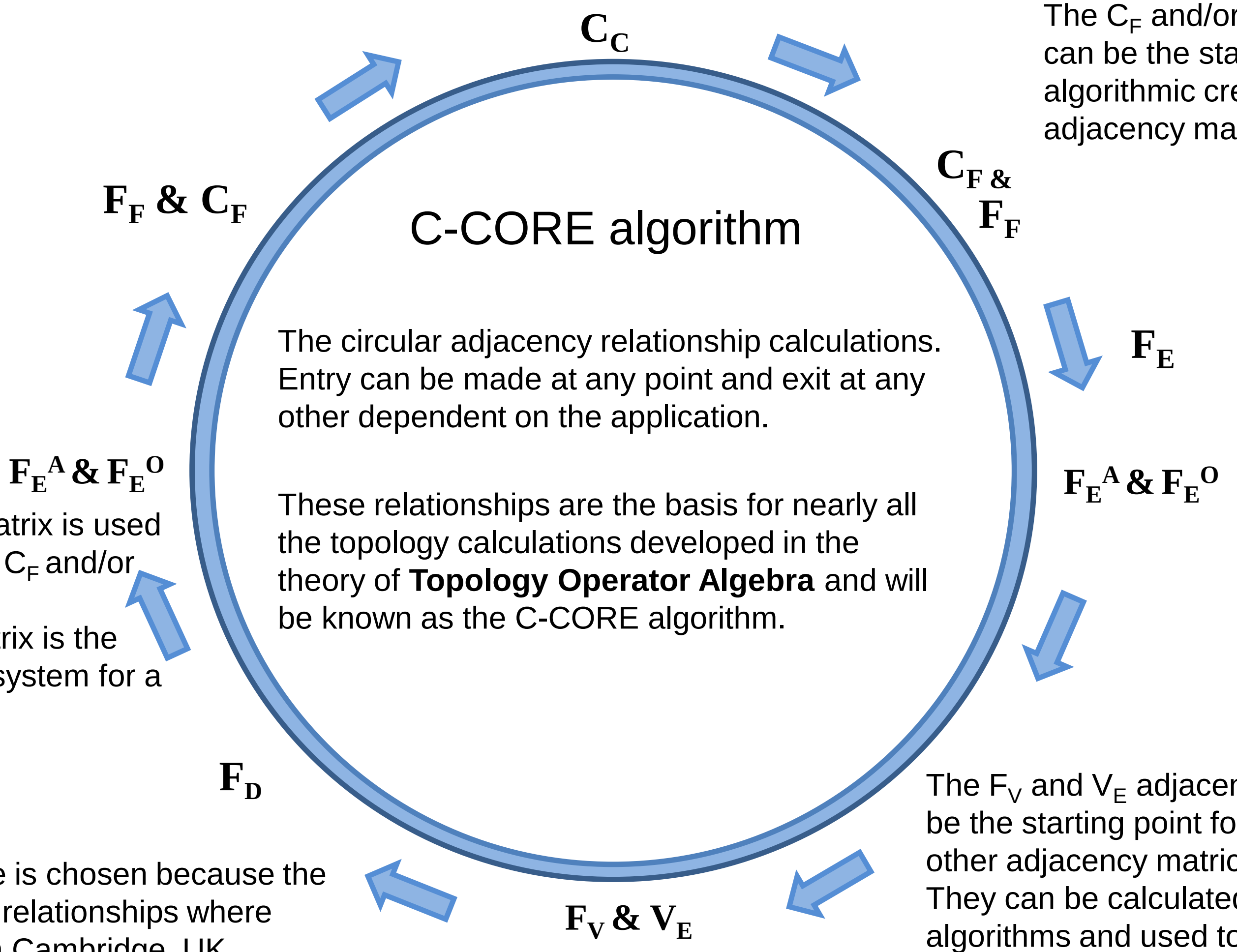
The mathematical principles described here are independent of the application(s) where the ideas have originated. There are a number of other applications where the same mathematics can be used to create shape information that so far has been obscured or missing.

An example: VRML 1. 0 & 2. 0 file formats describe a model using the point coordinates and the F_v and V_E matrices, but does not contain the cell definition. Here is a method to calculate the cell topology when importing a VRML file.

The General Theory of Representation and Manipulation of Shapes

The C-Core algorithm

The C_F and/or F_F adjacency matrix can be the starting point for an algorithmic creation of the other adjacency matrices.



The circular adjacency relationship calculations. Entry can be made at any point and exit at any other dependent on the application.

These relationships are the basis for nearly all the topology calculations developed in the theory of **Topology Operator Algebra** and will be known as the C-CORE algorithm.

The C-Core algorithm is a key component in the Operators described later.

The F_E^A adjacency matrix is used for calculating the F_F , C_F and/or C_C matrices

The F_E^O opposite matrix is the A_e+A_i in the equation system for a cell

The F_V and V_E adjacency matrices can be the starting point for the creation of other adjacency matrices.

They can be calculated in other algorithms and used to create the F_F , C_F and/or C_C matrices.

The C-CORE name is chosen because the discovery of these relationships where made while living in Cambridge, UK.

The General Theory of Representation and Manipulation of Shapes

The relationships between the adjacency matrices – try it out!

The relationships between adjacency matrices opens up a new world of opportunities. But don't take my word for it: TRY IT OUT!

The opportunity is to use the C-CORE algorithm to:

- Expand the algorithm to cellular clusters, i. e. , to create a wireframe network that can automatically be converted to a set of faces that bound a cluster of meshable cells, for substantial reduction in manual effort. An additional algorithms to find the Cells in the F_F matrix is needed to create the meshable cells.
- Try it out on a number of other topologies starting from different stages in the development of the adjacency relationships.

Then use the C-CORE algorithm to:

- Create engineering tools straight away within an FEA pre-processor (hexahedral meshes are still made manually by building the cellular models from a wireframe structure, a time consuming and error prone effort). The C-CORE algorithm can create cells from a wireframe in a flash.

The C-CORE algorithm description is included so others can implement it and demonstrate to themselves two things:

- This is not too difficult for them to get involved;
- It works, hence confidence can be built in the introduction of a completely new approach to handle solid models.

The C-Core algorithm, the circular calculation of the relevant adjacency matrices is a mathematical principle that cannot be protected as Intellectual Property in terms of patents (maths is one of five exceptions that cannot be covered by a patent).
However, these relationships are CORE functionality in a large number of algorithms (processes) for manipulating shapes in computer software based on Topology Operator Algebra, and most, if not all, of these can be protected Intellectual Property, as either patents, trade marks, design rights or company secrets.

The General Theory of Representation and Manipulation of Shapes

General topology clusters

General polyhedral clusters

General cellular non-manifold B-rep clusters

Tessellation process

3D Kernels

Aw – I L = 0

W = [.......]

Meshing process

Coarse Hexahedral Cluster

Hexahedral Mesh

The Special Theory of Shape Representation and Manipulation

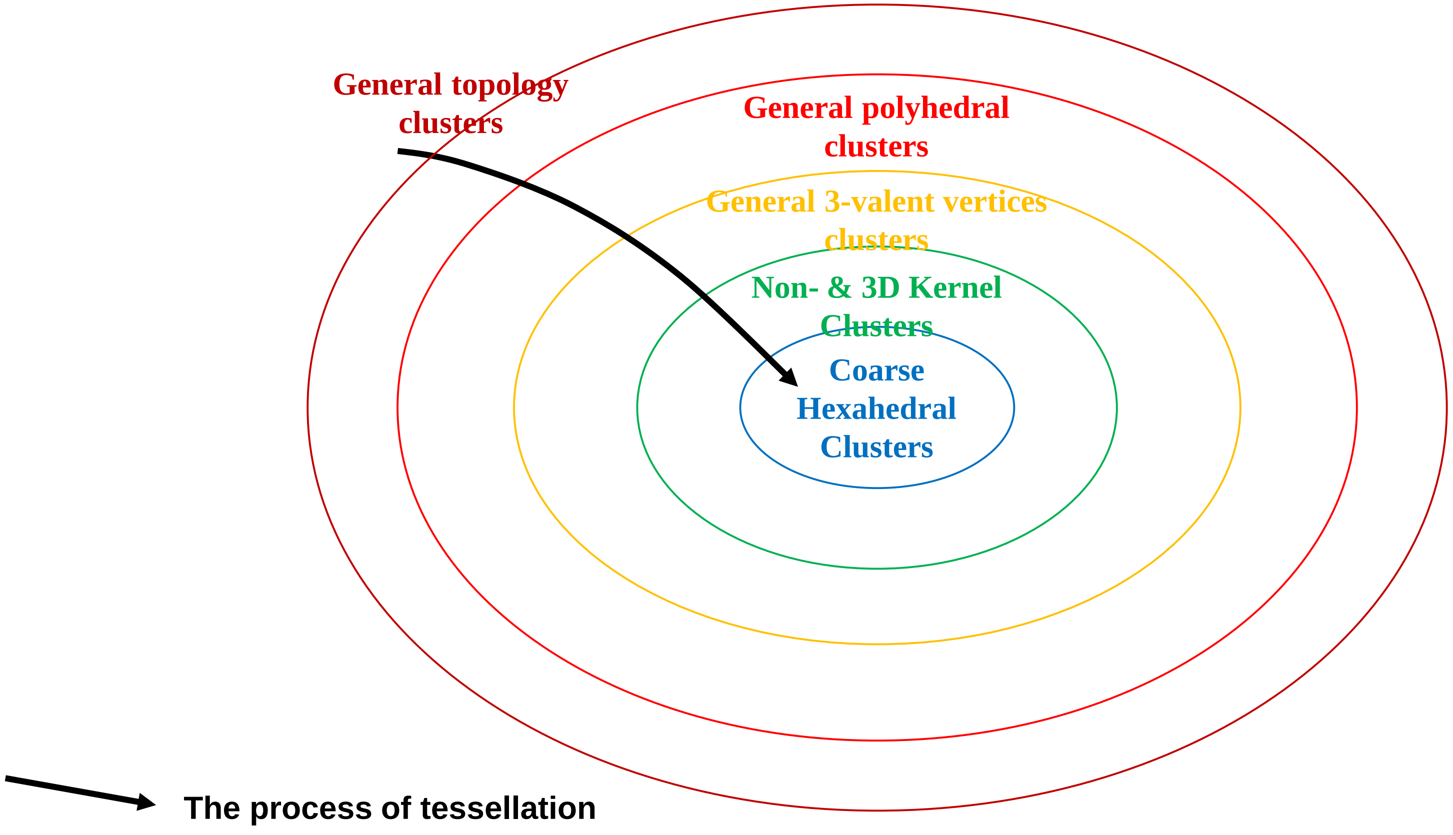
The General Theory of Shape Representation and Manipulation

The unification of Tessellation and Meshing in one mathematical theory

The Special Theory of Shape Representation and Manipulation corresponds to traditional meshing technologies where in addition “meshable shapes” have to be created manually or semi-automatically.

The General Theory of Shape Representation and Manipulation includes the Special Theory, but expands it into the realm of general shapes where Operators can tessellate them into clusters of smaller shapes, eventually end up with a cluster of 3D Kernels.

The General Theory of Representation and Manipulation of Shapes

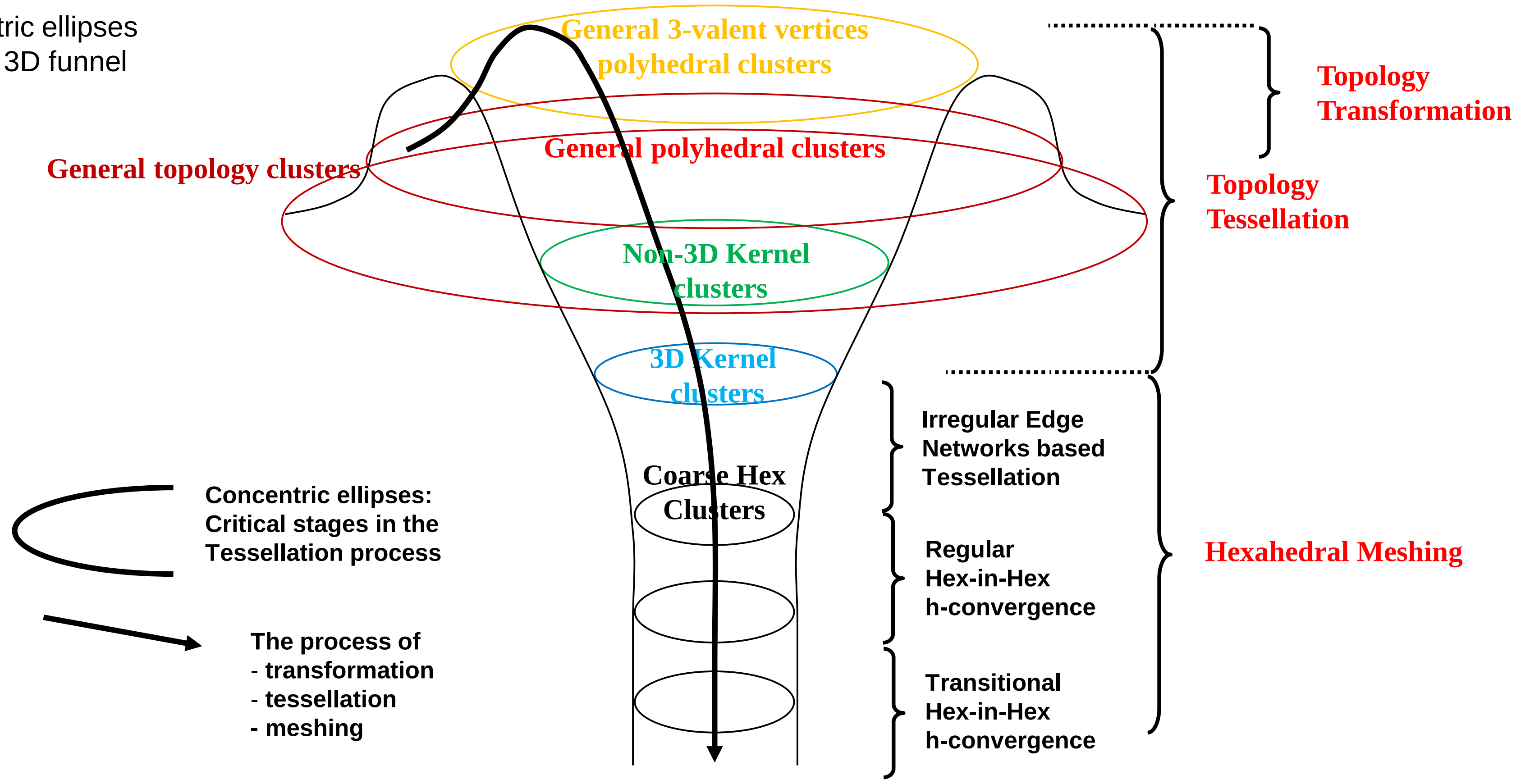


Successive tessellation for automatic hexahedral meshing uses a particular set of operators and types of topologies, all true subsets of more complex topologies, to create a path to a Coarse Hexahedral Cluster. The tessellation Operators transform a cluster of cells to another cluster of cells, each of the clusters consists of cellular non-manifold b-rep solid models.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

The General Theory of Representation and Manipulation of Shapes

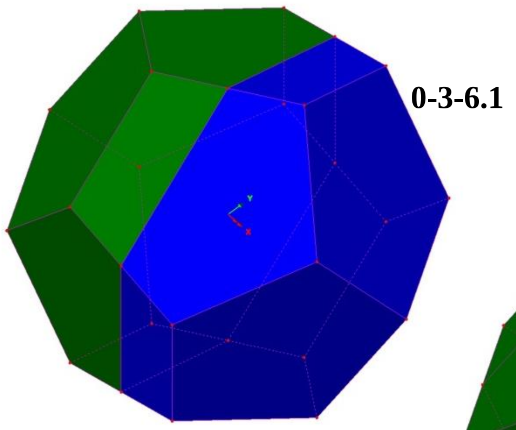
The concentric ellipses are in fact a 3D funnel



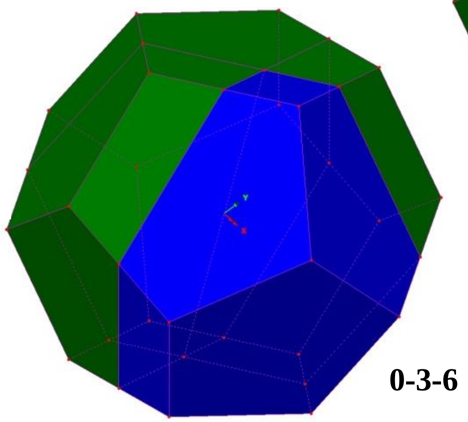
The starting point is general topology clusters (a cluster may consist of one cell), which may not satisfy the requirement of connectivity so essential to the topology based algorithms such as Dislocation Meshing.
Hence the Topology Transformation process is introduced to improve the suitability of the shapes to the tessellation and meshing processes.

The General Theory of Representation and Manipulation of Shapes

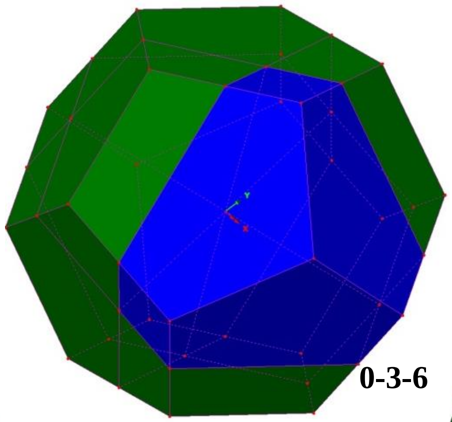
Examples of Topology Operators



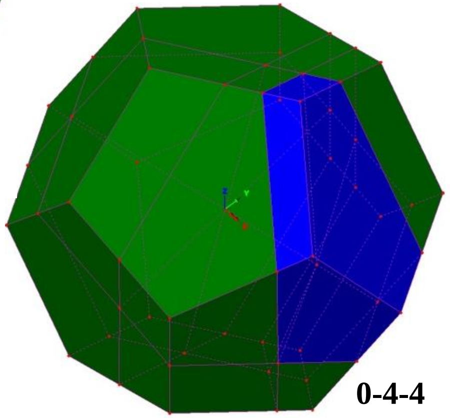
A sub-cell coming out of the first planar Operator



A sub-cell coming out of the second planar Operator



A sub-cell coming out of the third planar Operator



A sub-cell coming out of the fourth planar Operator

For a given sub-set of planar Operators in a group there is a particular tessellation into 3D Kernels, the order in which they are applied is of no significance, i. e. , they are commutable. Any combination of planar Operators within a group fits into the circle shown for the C-Core algorithm.

Each of the cells in a cluster of cells have their own F_F matrix that can be extracted from the combined F_F matrix coming out of the Operators. For clusters and individual cells the entire set of matrices, including the F_C and C_C adjacency matrices, can be computed through linear algebra.

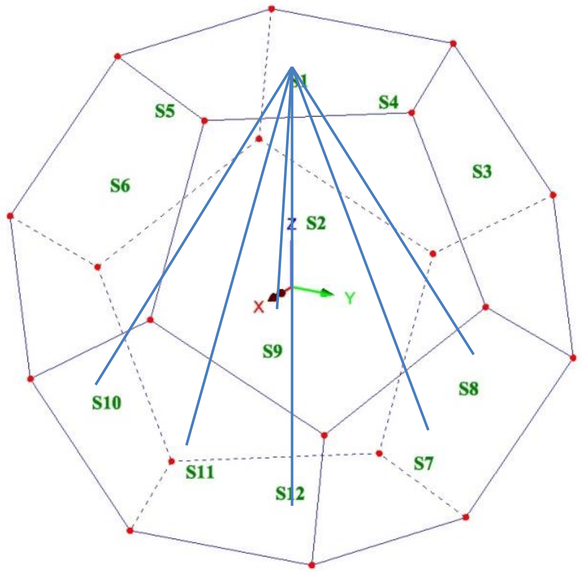
The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators

F_F ⇒ The matrix representing irregular edges connecting two faces only

```
1 1 1 1 1 1 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
. . . . . . 1 1 1 1 1 1 . . . . . . . . . . . . . . . . . . . . . . . .
. . . . . . . . . . . . 1 1 1 1 1 1 . . . . . . . . . . . . . . . . . .
. . . . . . 1 . . . . . . . . . . . 1 1 1 1 1 . . . . . . . . . . . . .
. . . . . . . 1 . . . . 1 . . . . . . . . . . 1 1 1 1 . . . . . . . . .
. . . . . . . . . . . . . 1 . . . . 1 . . . . . . . . 1 1 1 1 . . . . .
1 . . . . . . . . . . . . . . . . . . 1 . . . 1 . . . 1 . . . 1 1 . . .
. 1 . . . . . . 1 . . . . . . . . . . . . . . . 1 . . . 1 . . . . 1 1 .
. . 1 . . . . . . 1 . . . . 1 . . . . . . . . . . . . . . 1 . 1 . . . 1
. . . 1 . . . . . . 1 . . . . 1 . . . . 1 . . . . . . . . . . . 1 1 . .
. . . . 1 . . . . . . . . . . . 1 . . . . 1 . . . 1 . . . . . . . . 1 1
. . . . . 1 . . . . . 1 . . . . . 1 . . . . 1 . . . 1 . . . 1 . . . . .
```

The irregular edges connecting the top face S1
to six other faces as shown on the right



A complementary set of Operators can be used to tessellate a cell:
The irregular edges connecting pairs of pentagonal faces must be at least a face apart, i.e., each face has five candidate faces plus the opposite face, six cases in all.
This can be expressed as an incidence matrix as shown above. There are 72 perturbations, but due to the duality, there are only 36 entries in the matrix:

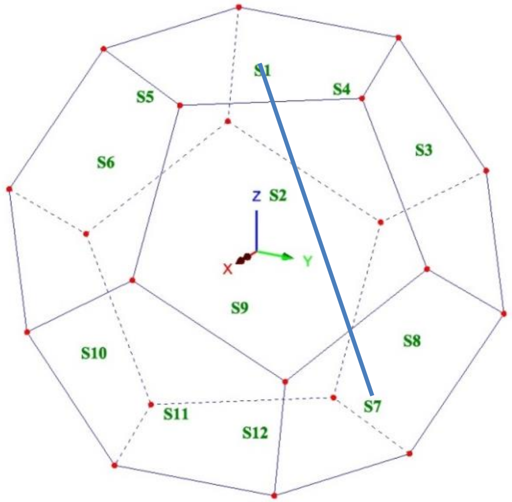
- 30 entries for adjacent plus one faces;
- 6 entries for adjacent plus two faces;

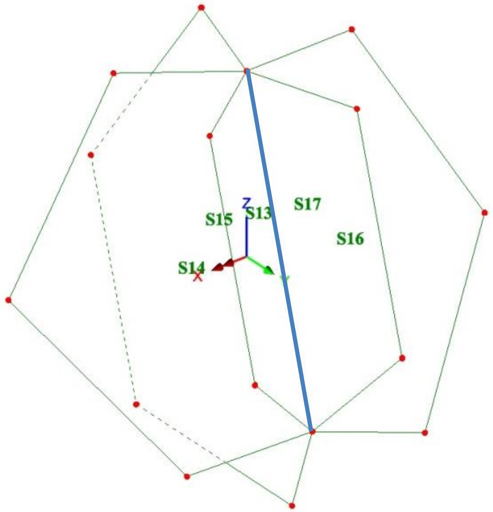
Each of the faces have 6 connections. The matrix above can be derived directly from the F_F matrix.

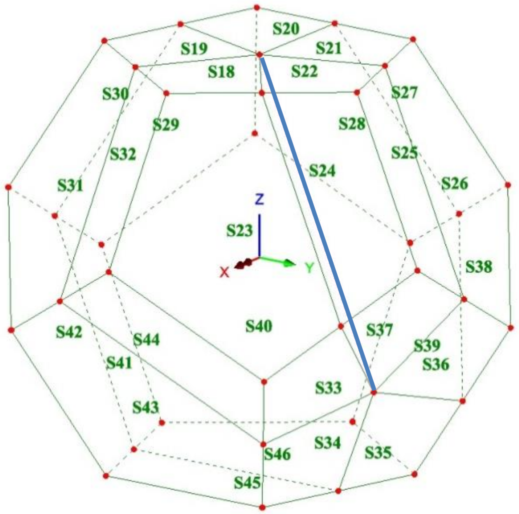
Each of the irregular edge vectors can be used to create a tessellation, i.e., as an Operator as shown in the following.

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators







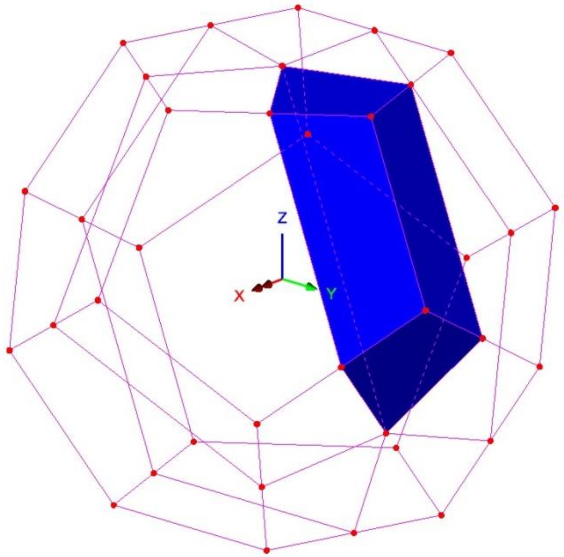
$$\begin{bmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

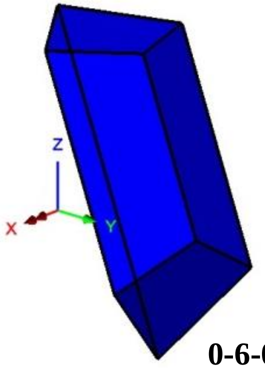
An Operator can be created to tessellate any object using a five way irregular edge connecting two faces.
It is given the name "Single Edge Pentagonal Operator".
This is one a several possible operators, and is chosen here because it matches the two faces and their irregular vertices to be connected.
The Operator created from the irregular edge between faces S1 and S7 tessellate the 0-0-12 into a set of five cells, their faces are shown in the picture to the right. The resulting sub-cells are shown in the subsequent pages.

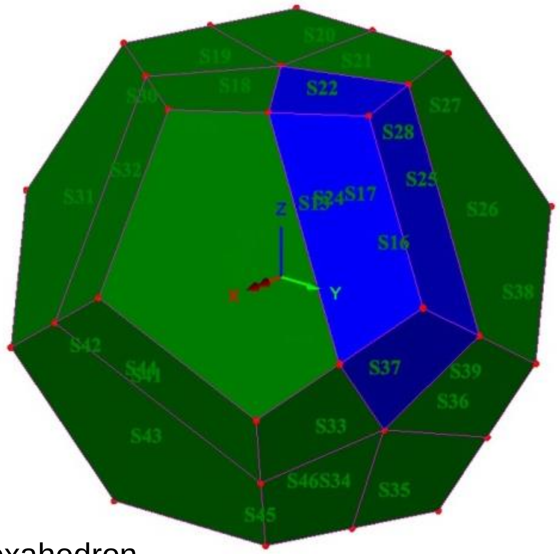
The irregular edge connecting S1 to S7, i.e., the leftmost of the 36 in the connectivity matrix developed in the previous slide

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators







The first sub-cell is a 0-6-0: a Hexahedron

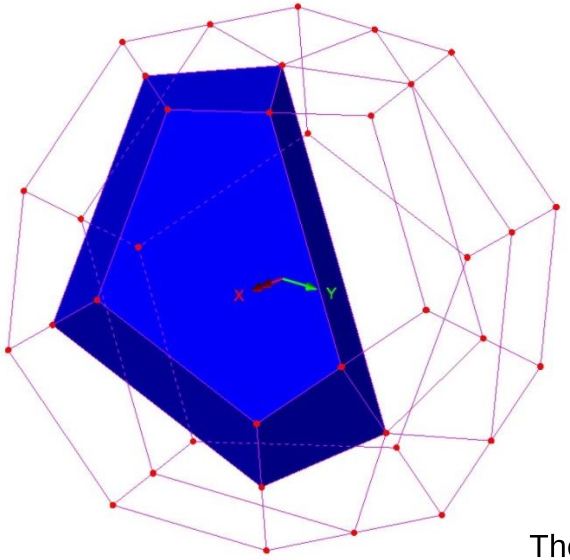
The operator defines the tessellation of the faces that are hit by the internal faces, creating a set of cutting edges for each of the external faces. Each external face is tessellated and the C-Core Algorithm is then used to compute the adjacency matrices and the other relationships leading to the compound F_F matrix, containing all the sub-faces for the assembly, external sub-faces as well as the internal faces.

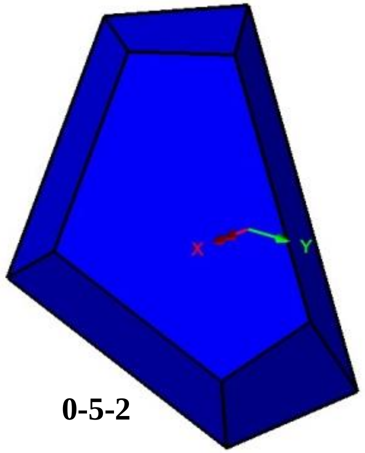
Each of the individual F_F matrices are then extracted to define the sub-cells that make up the original object.

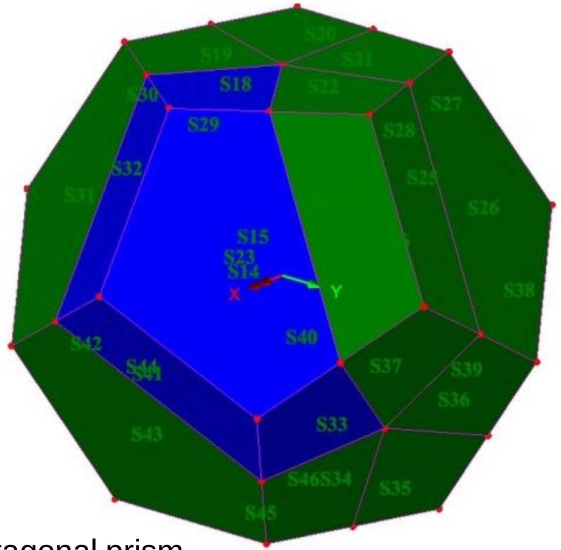
The faces in the operator and the external faces of the shape are used to define the topology of the sub-cells, ensuring that the connectivity across the cell boundaries is maintained for mesh connectivity.

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators







The second sub-cell is a 0-5-2: a pentagonal prism

The C-CORE algorithms is used in the computation of the sub-cells coming out of the Operator. Each cell is found as a loop of faces forming a closed set of faces, i.e., an F_F matrix.

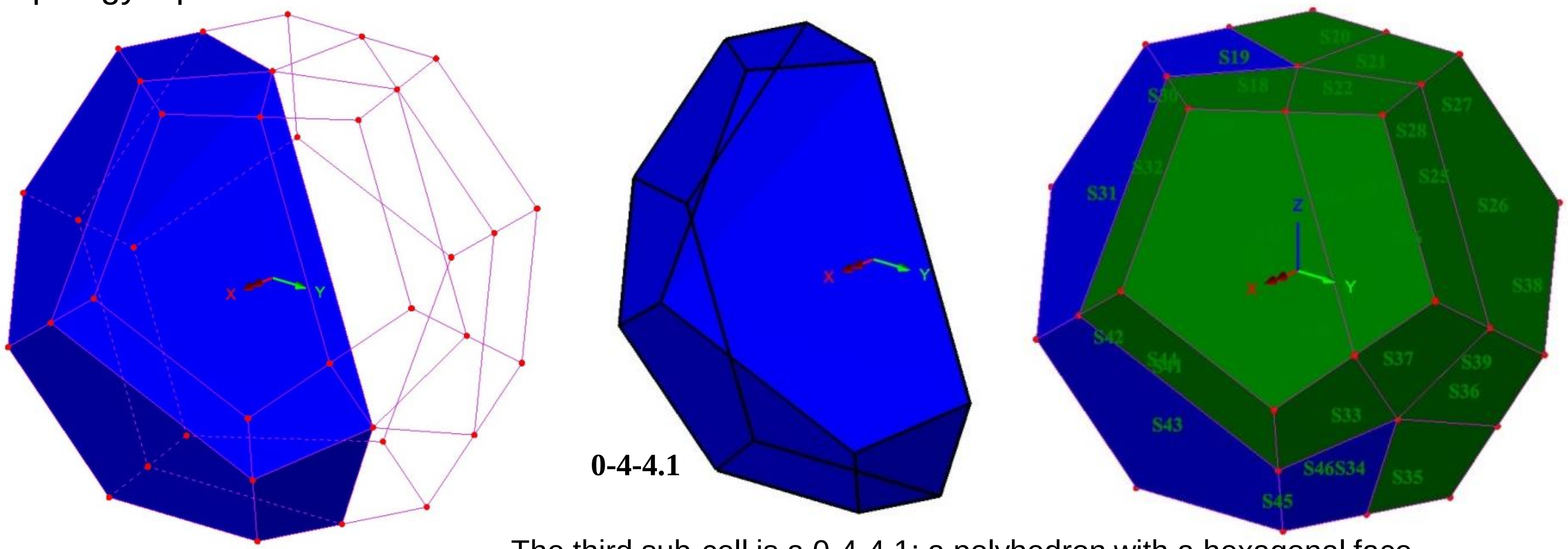
The cells created using this particular operator are partly 3D Kernels and partly not. Additional tessellation using other operators is needed to create a cluster of only 3D Kernels.

DISLOCATION MESHING –

A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators



The third sub-cell is a 0-4-4.1: a polyhedron with a hexagonal face

There are twelve irregular vertices in the exterior faces of a 0-0-12, one in each of the twelve pentagons. The operator demonstrated here connects two irregular vertices in faces a face apart. Similarly, pairs of irregular vertices can be connected by individual irregular edges as defined by the operator. There are 36 combinations of irregular edges all together, each forms the basis for subdividing a Dodecahedron into five sub-cells.

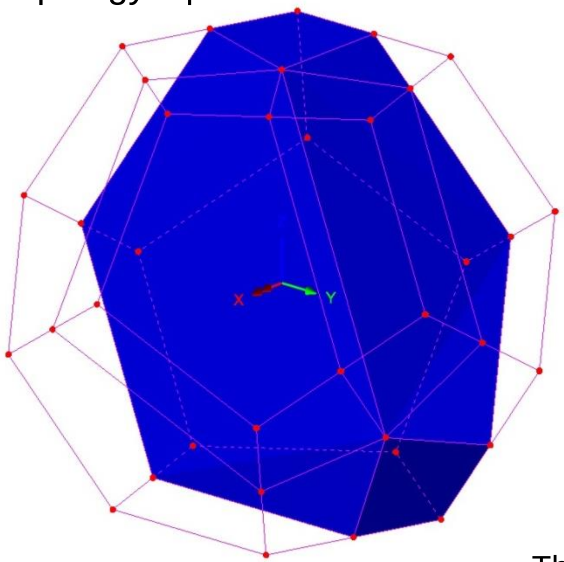
However, there are other ways of connecting irregular vertices on the exterior of a polyhedron, most of which will not lead to a Coarse Hexahedral Cluster. The development of 3D Kernels introduces restriction of what is a useful set of irregular edges in combination. These sets are expressed in the G-matrix for every equation system.

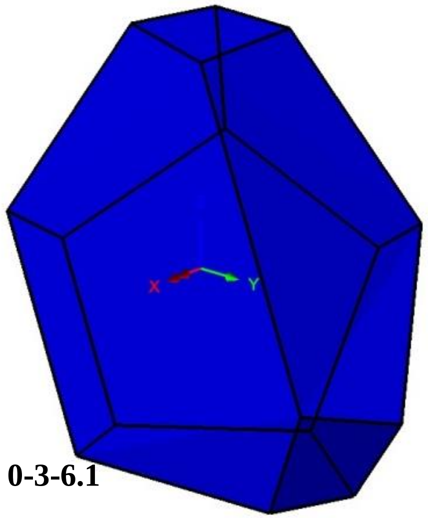
The operator demonstrated here is one of many that can be used to tessellate any shape into simpler cells represented as a cellular non-manifold boundary-representation solid model.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

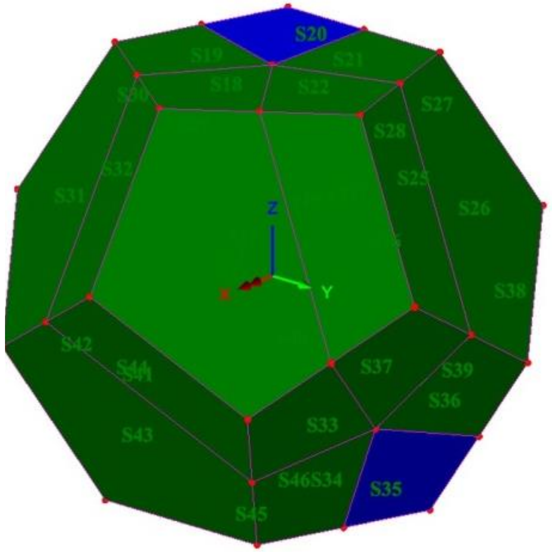
The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators





0-3-6.1



The forth sub-cell is a 0-3-6.1: a polyhedron with a hexagonal face

The cluster of cells can be subdivided into a coarse hexahedral cluster, which again can be meshed using traditional Hex-in-Hex meshing algorithms creating either a regular mesh or any transitional mesh.

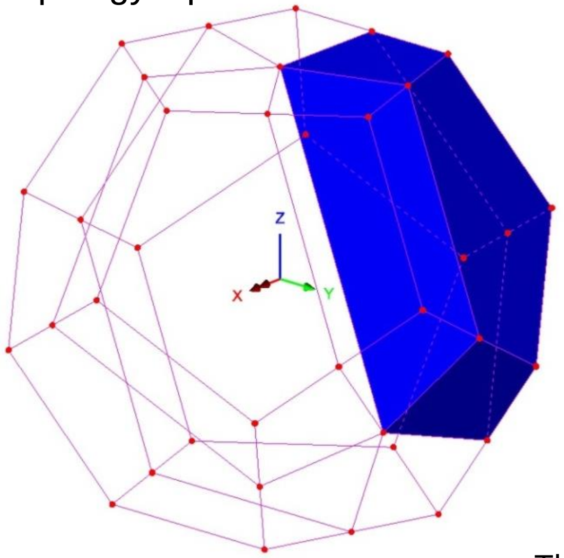
There is more to the tessellation and meshing than what is outlined here, but the key message is that operators working on adjacency matrices and other relationship matrices can create the meshable shapes that traditional meshing can use as input in a deterministic algorithm.

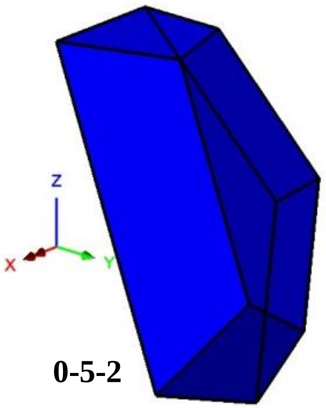
The mathematical representation for these operators and their results are under development, but the structure to the approach is clear: linear algebra representation of the topology of a shape can be used to model tessellation and meshing.

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

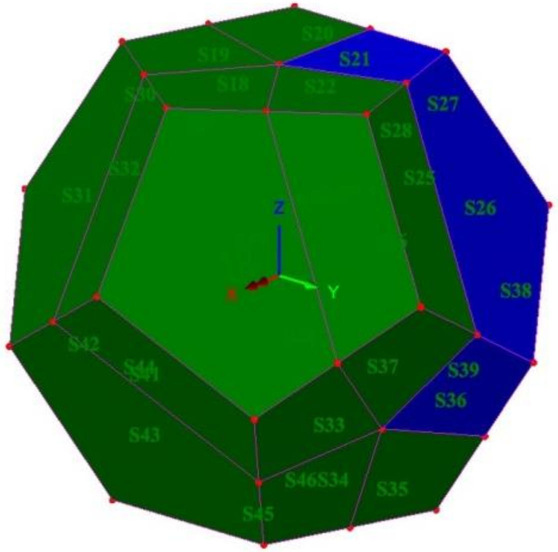
The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators





0-5-2



The fifth sub-cell is a 0-5-2: a pentagonal prism

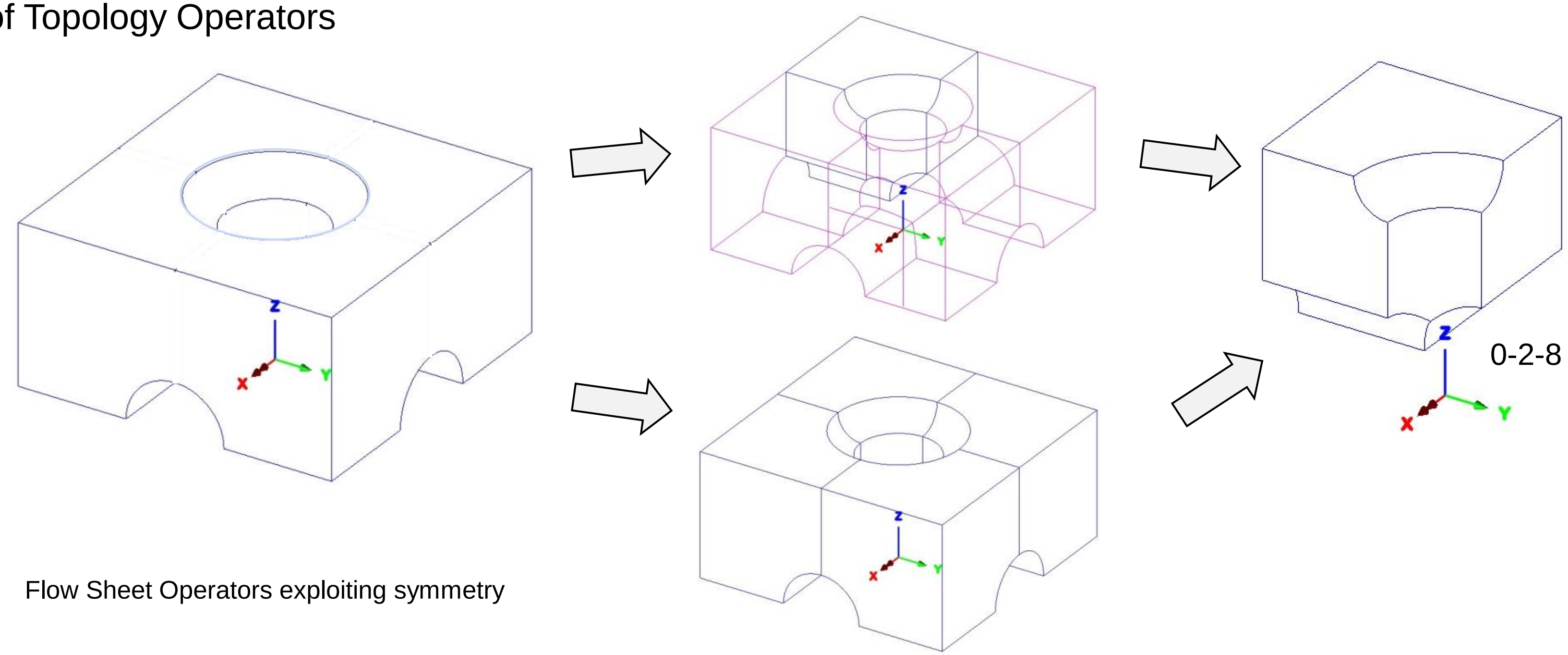
For a Full Separation tessellation into hexahedra, six of the total of 36 irregular edges need to be combined to subdivide the Dodecahedron into sub-cells. Why six? Because there are twelve faces and six edges combine all faces into pairs. The challenge is to find which six of the total of 36 form a group, i.e., a row in the G-matrix.

The five sub-cells created by applying the first Operator can be the input to the second Operator, and the result from the two Operators can be the input to the third and so on until all six have been applied. Apart from the use of single five-way irregular edge Operators, any combination of any subset of the six Operators will create a cluster of 3D Kernels. After all six have been applied, the result is a cluster of Prisms, all 3D Kernels, which can be tessellated using the MPSD algorithm as an Operator and create the Coarse Hexahedral Cluster.

The order in which the Operators for a particular tessellation is applied is of no significance, i.e., they are commutable.
Each of the cells in a cluster of cells have their own F_F matrix that can be extracted from the shared F_F matrix coming out of the Operators.

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators



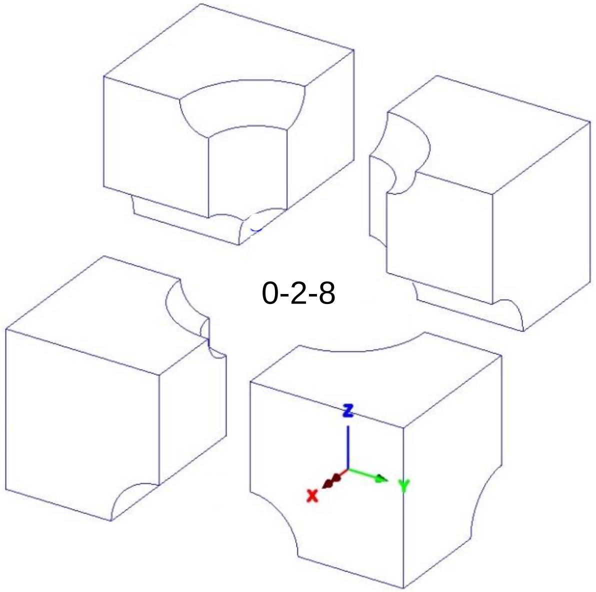
Flow Sheet Operators exploiting symmetry

A general shape can be tessellated into a cluster of less complex cells using planar flow sheet Tessellation Operators and any of the three-way, four-way and/or five-way Operators, or any other combinations of irregular edge networks. The use of irregular edge network Operators is demonstrated when creating the underdetermined super set equation system.

Symmetry in a shape is often a dominant characteristic of the shape, and flow sheet Operators can be automatically positioned to carry out a tessellation into symmetrical parts. Subsequent subdivision may use the symmetric nature of the cluster of cells to create identical Coarse Hexahedral Clusters. However, the distribution of loads and boundary conditions may not be symmetrical, so alternative tessellations may be better for representing the unsymmetrical deformation/stress fields. The equation system for each of the 3D Kernels contains a large number of alternative irregular edge networks, hence will allow alternative meshes for each cell in the cluster as long as the meshes are identical at their boundaries.

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators



Cellular Non-manifold Cluster

$$\begin{bmatrix} A_e \mid A_i \end{bmatrix} \begin{bmatrix} w_e \\ w_i \end{bmatrix} - \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} L \end{bmatrix} = 0 \qquad \begin{bmatrix} G \end{bmatrix}$$

0-2-8

$$\begin{bmatrix} A_e \mid A_i \end{bmatrix} \begin{bmatrix} w_e \\ w_i \end{bmatrix} - \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} L \end{bmatrix} = 0 \qquad \begin{bmatrix} G \end{bmatrix}$$

0-2-8

...

0-2-8

$$\begin{bmatrix} A_e \mid A_i \end{bmatrix} \begin{bmatrix} w_e \\ w_i \end{bmatrix} - \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} L \end{bmatrix} = 0 \qquad \begin{bmatrix} G \end{bmatrix}$$

0-2-8

Four in all

$$\begin{bmatrix} A_e \mid A_i \end{bmatrix} \begin{bmatrix} w_e \\ w_i \end{bmatrix} - \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} L \end{bmatrix} = 0 \qquad \begin{bmatrix} G \end{bmatrix}$$

Equation System for the Assembly

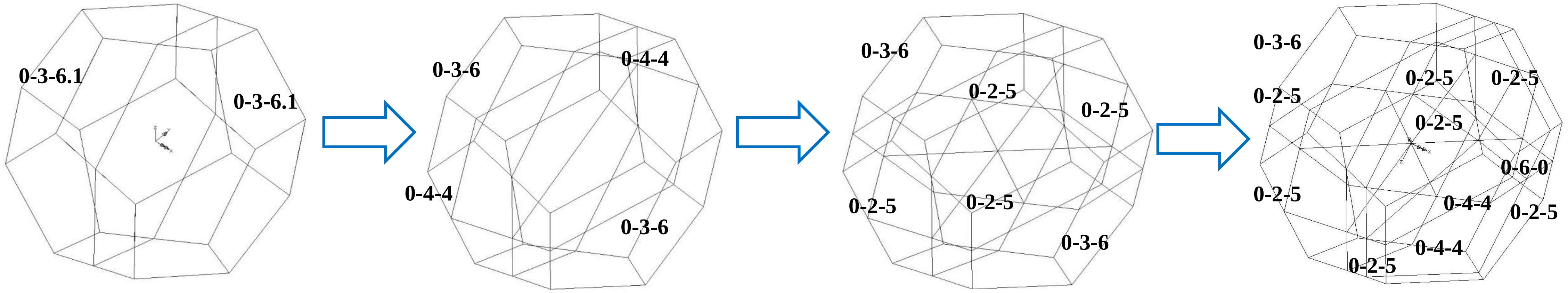
A combined equation system for clusters is based on using the individual equation system for the cells as input to an assembly algorithm bringing them together into a single equation system. The assembly algorithm will use the knowledge of how the various cells share boundary with other cells, and how the internal flow sheets described in the G-matrices will propagate into the other neighbouring cells. Additionally, the faces with no irregular vertex will add additional flow sheets, as the irregular edge networks on either side will need to be positioned relative and absolute to each other.
Each of the equation systems may have a complete (Full Separation) or reduced (irregular edges interacting with each other) G-matrix, depending on how the evaluation of which of the cases to include has eliminated irrelevant cases prior to assembly.

There is no upper limit to the number of cells that can be assembled, nor the number of flow sheet groups to be included, all in one equation system.

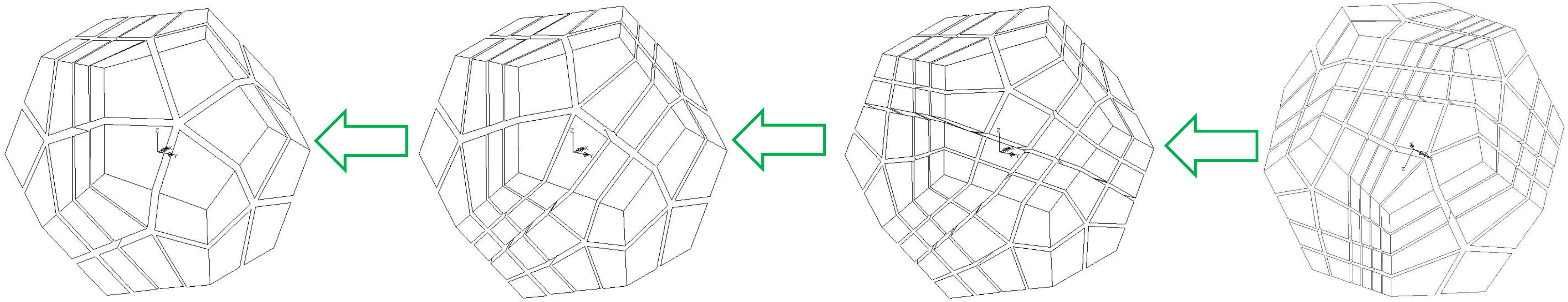
DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators



Successive use of Flow Sheet Tessellation Operators

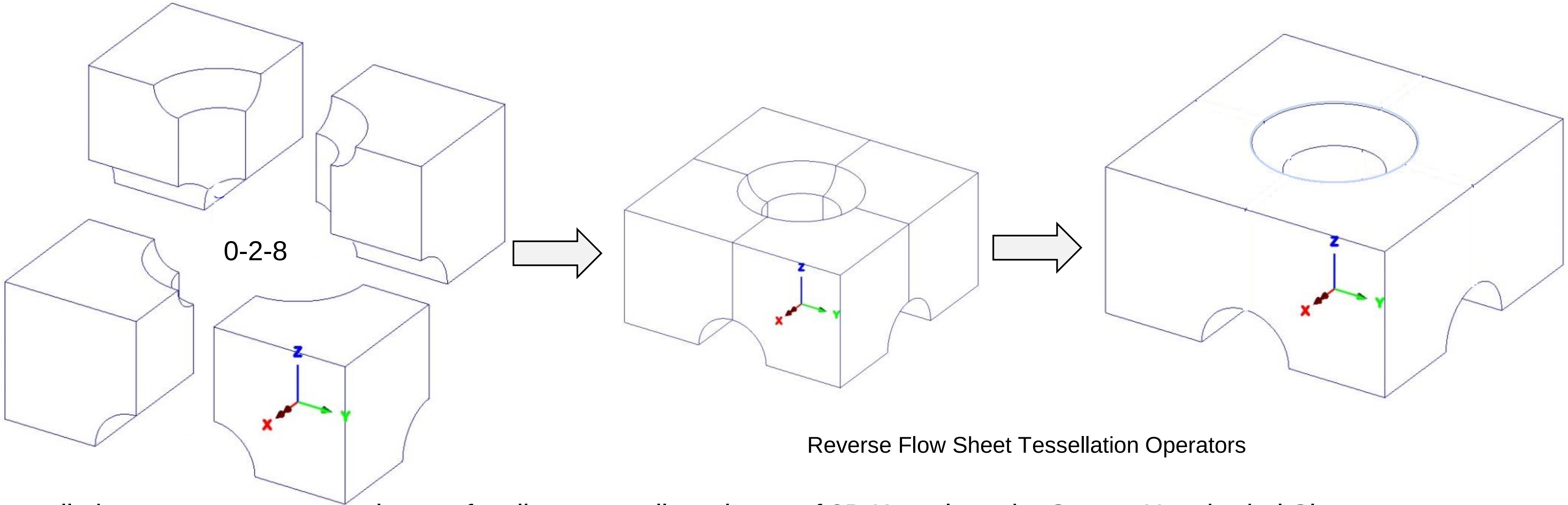


Successive use of Merge Operators, i.e., Reverse Flow Sheet Tessellation Operators

The Reverse Flow Sheet Tessellation Operators can work on the CHC or the flow sheet representation of the cluster of cells.

The General Theory of Representation and Manipulation of Shapes

Examples of Topology Operators



Reverse Flow Sheet Tessellation Operators

So far, the tessellation process creates a cluster of cells, eventually a cluster of 3D Kernels and a Coarse Hexahedral Cluster.
The reverse process will merge a cluster of cells into a more complex cluster of cells, successively increasing the topological complexity of the members of the cell cluster until eventually the cell cluster consists of the design model.

The Operators used to tessellate have an equivalent set of merge Operators:

- Flow sheet Operators have an equivalent Merge Operator which combine a set of single sided faces from two cells into a double sided internal face.
- Each Irregular edge network Tessellation Operator has an equivalent Merge Operator identifying a set of external faces for several cells into an internal face separating the irregular edge networks.

So, the process of merging the simpler cells into more complex ones will bring with it the external boundaries between the sub-cells as internal flow sheets on the next level, showing how the Operators cut through a shape from the level below. Internal flow sheets can be included or excluded for merging at the next level.

A Mathematical Continuum

There are a large number of possible Operators that can be used to manipulate the topology representation of a cluster of cells.

- Any single or group of Flow Sheet(s) can be an Operator;
- Any single or group of irregular edge network(s) can be an Operator;
- Any single or group of 3D Kernel(s) can be an Operator;
- And more …

The adjacency relationships described in an incidence matrix for the Faces in a cluster of cells, will contain all the F_F matrices for the cells in one big matrix. There are a number of other adjacency matrices used in the process and a number of other matrices used for other purposes.

The mathematical representation is Linear Algebra and Transformations on the matrices.

A mathematical representation of a Cellular Non-Manifold Boundary-Representation Solid Model is the same as the individual element in an FEA mesh, and results related to the nodes and elements calculated in the analysis. FEA elements have specific shape and their own numbering systems. These are compatible with the adjacency matrices used for representing solid model topology.

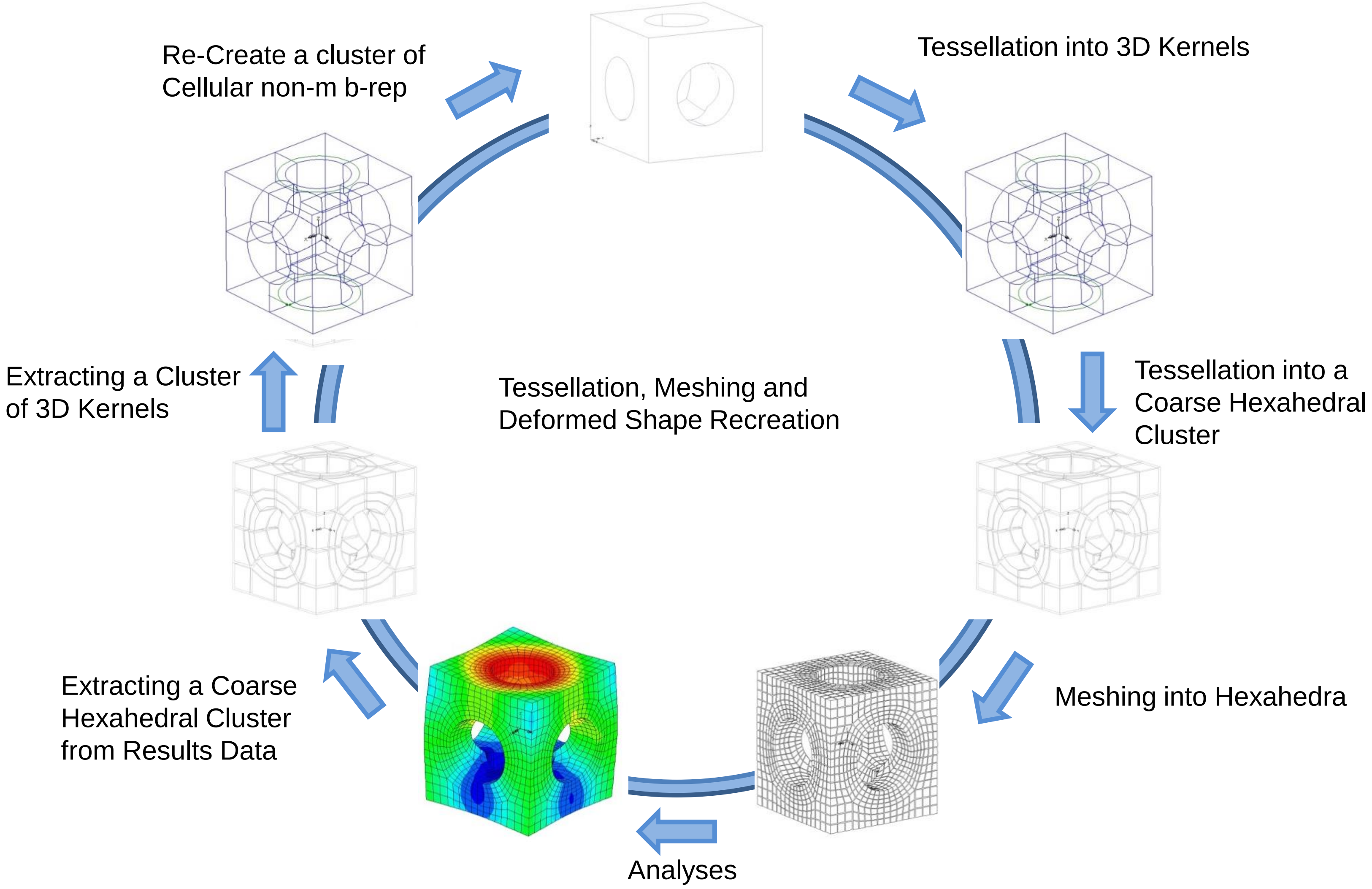
The Operators to Tessellation and Meshing are the same from top to bottom, it is a matter of choice when the adjacency relationship matrices are replaced by the traditional nodes and elements representation of a mesh. (This is an intriguing thought, being flexible in setting the boundary for when to switch, i.e., defining the point of “meshability” … However, it adds another dimension in complexity …)

The reverse process, using Merge Operators to create continuous representations from a discrete FEA mesh and subsequent cell clusters all the way to the top, closes the loop. The mathematical representation described here will enable you to take your solid model all the way to FEA results and back, creating a deformed solid model for the load case and boundary conditions you consider. And importantly, a uniform mathematical representation is used in all stages.

Question: Can you imagine what you would use it for?

DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

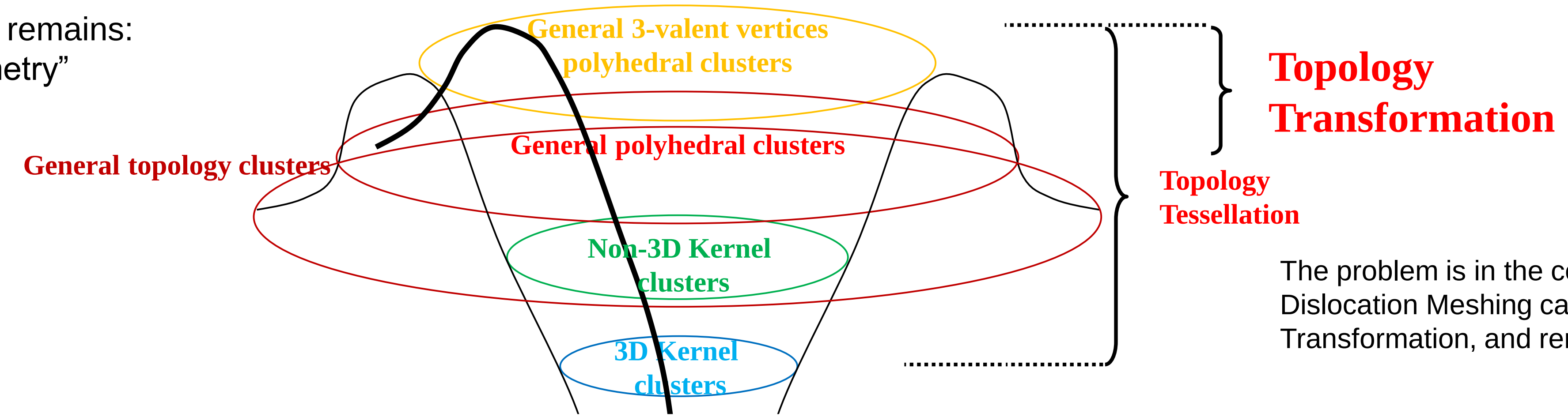
A Mathematical Continuum



DISLOCATION MESHING –
A CREDIBLE SOLUTION TO AUTOMATIC HEXAHEDRAL MESHING

A Mathematical Continuum

An obstacle remains:
“Dirty Geometry”



The problem is in the context of Dislocation Meshing called Topology Transformation, and remains unsolved.

Even experienced users of Solid Modelling Systems have a tendency to create faulty geometries. Pictures of the shapes look perfectly OK, but the models are often lacking integrity. That is, the geometry and topology may be incomplete, have gaps and double lines, very thin slivers between surfaces, etc. This is known as “dirty geometry” and causes major problems for any application down-stream from CAD model creation, such STL-based 3D printing, CNC machining and yes, indeed, FEA tessellation and meshing.
In addition, two incompatibility problems remain:

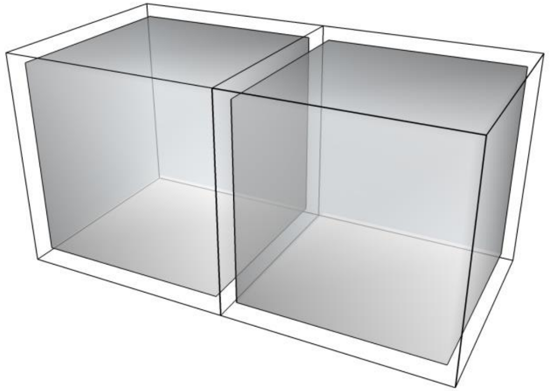
- The general topology found in the design model is incompatible with the topology needed for meshing algorithms to deliver quality meshes;
- The geometry representation used in the design system may be incompatible with the geometry in the down-stream application.

The integration of CAD-FEA remains at the same level as it was 35 years ago, when I first got involved as a CAD-FEM Engineer.

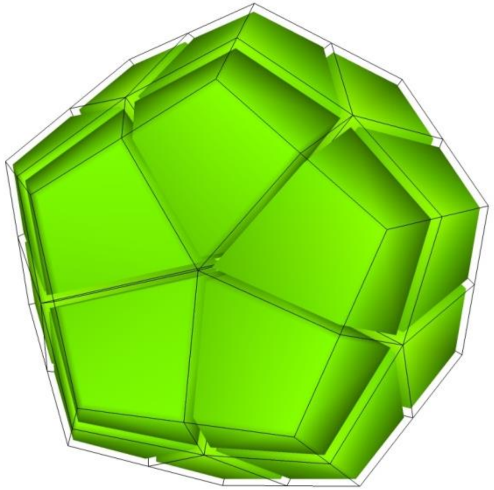
A lot of non-value added work is spent correcting mistakes in CAD-models outside the design systems themselves, mistakes that should not have been made in the first place. All down-stream application have software tools to detect and correct “dirty geometry”, and an industry has mushroomed making a killing from fixing problems that should not exist.

An MPSD mesh of a Dodecahedron created in SketchUp 2017

A Mathematical Continuum



Two adjacent 0-6-0 cells in Sketchup 2017



The Dodecahedron tessellated to CHC using MPSD presented in Sketchup 2017

The way forward consists of a three-pronged approach:

- **Don't transfer shapes to another computer application.** Develop the mathematical representation of shapes in the CAD system so it can be used for design and analysis. Then migrate the analysis modelling technology, including meshing, into the design system so analysis becomes a solid modelling function. The problem of data exchange, system integration and shared shape information is solved, because data exchange is eliminated.
- **Train engineering analysts in the use of the design system** for shape modelling, augmentation, meshing, analysis and result presentation. Train them to create correct models first time. Share models with designers within the same CAD-system.
- **Develop better tools for design and analysis model creation.** The down-stream correction technologies must be replaced by in-system model checking procedures, ensuring that modelling mistakes are identified and corrected before the models are used for other applications. However, the problem will only be solved when the creation process has topology verification, manipulation and control embedded in the creation stage. The General Theory for Representation and Manipulation of shapes has another application area to address.

What about Formulation, Verification & Validation?

Verification	Validation
Making sure it works	Making sure it does what it's supposed to do.
Getting the math right.	Getting the physics right.
Providing an accurate FE analysis.	Checking the FEA against tests.

This text and the subsequent pictures are taken from a web-site: http://www.deskeng.com/de/verification-vs-validation/ , material added March 1 2015.

Good working practices already include Verification and Validation as a part of the analysis QA process, often under a different name.
A selected cut and glue abstract from the web-side http://www.deskeng.com/de/verification-vs-validation/ includes:

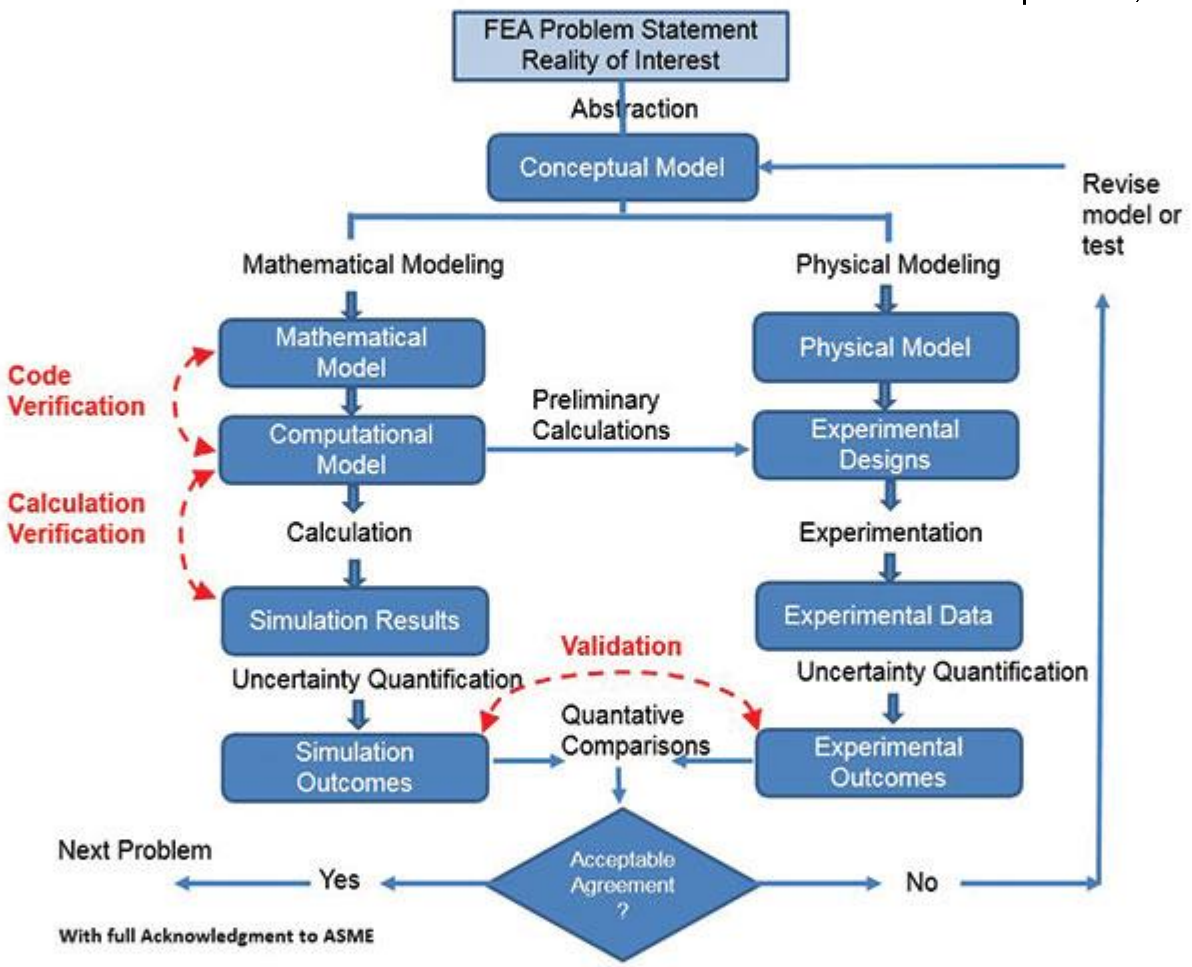
"Uncertainty Qualification
We have methods of assessing mesh convergence to give confidence in FEA local stresses, but they are ad hoc. A simple automated metric can mislead here, without an understanding of load paths and stress distribution. The challenge to a good understanding of why stresses occur is that it is difficult for anyone, other than an expert, to picture. This is where FEA post-processing lets us down.
There is a set of tools that could be developed to allow us to understand the results in a way which is required for effective verification.
…
A further shortfall of current FEA practice is the assessment of uncertainty over boundary conditions and loading. Accurate modelling of these aspects can be difficult. We can carry out sensitivity studies on loading levels and line of action, boundary stiffness and others,
but it is tedious to do this manually. Again, **if we are serious about V&V as an industry, we need FEA tools to facilitate this**."

Dislocation Meshing can be implemented in an Engineering Workbench to fill in the gaps described here …

What about Formulation, Verification & Validation?

Verification and Validation in FEA

A flow chart showing how V & V relate in an FEA process, a V& V plan



A key requirement in a V & V environment is how easy it is to run many analyses to study the effect of changes.

By Sam Johnston - Created by Sam Johnston using OmniGroup's OmniGraffle and Inkscape (includes Computer.svg by Sasa Stefanovic) This vector image was created with Inkscape., CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=6080417

What about Formulation, Verification & Validation?

$$\frac{|\phi_i(u_{EX}) - \phi_i(uFE)|}{|\phi_i(u_{EX})|} \leq \tau_i$$

Exact solutions: in short supply

Approximate solutions: in plentiful supply

Where
- $\phi_i u_{EX}$ is the exact solution for a particular Result of Interest
- $\phi_i u_{FE}$ is the approximate solution from a numerical analysis for a particular Result of Interest
- i is an index for the number of Results of Interest for consideration, for example maximum displacement, temperature, stress, etc. at a particular point, curve, surface, volume
- τ_i is the relative error for the FE analysis compared with the EXact solution

There is an undisputable fact that the number of Exact Solutions is in short supply. A concerted effort in the simulation community should be undertaken to make full use of the existing theoretical work in abundance in classical mechanics literature and fill in the gaps that exist to create a database of examples to be used on a broad scale among the analysts committed to V & V working practices.

However, there will always be cases that do not have an equivalent Exact Solution and an alternative approximate solution is needed as a substitute. Dislocation Meshing has a role to play in this context, enabling engineers to create high quality meshes and models that can be used to establish a "near-exact solution" to compare against.

What about Formulation, Verification & Validation?

Dislocation Meshing is like a jet engine looking for a fuselage to fit around it. It fits into the Engineering Workbench idea, but needs more than just adding "more of the same" to state of the art CAD and FEA-systems.
The fuselage that looks most promising is Formulation, Verification & Validation.

Why include Formulation in front of Verification & Validation?
Because the issues of "Solving a Partial Differential Equation (PDE)" is not explicitly addressed in the traditional Verification & Validation process. However, it is a key aspect to getting reliable results, in particular for problems where dimensional reduction is used to simplify the analyses (3D shapes represented as membranes, plates and shells, and beams and bars)

Dislocation Meshing can offer a repeatable technique for convergence studies:

- Using h-convergence for a convergence study by increasing the width of flow sheets and moving irregular edge networks around;
- Analyses can be stream-lined in sequences by running for example VB Scripts for the purpose.
- However, comparing results from different analyses is still a cumbersome activity, as the relevant data has to be identified and transferred to Excel by any means available, often manual effort.
- Adopting established tools and methods from office automation including RDBMSs must come soon to engineering analysis.

- So, the use of Results-of-Interest marking in the input data and a post-process to store the results in a RDBMS, like Access and/or SQL-Server, will enable the analyst to assess effect of change in his analysis models.

Dislocation Meshing facilitates "many small analyses" as opposed to "one BIG analysis".
When the working methods surrounding Dislocation Meshing is stream-lined and can make good use of HPC,
The use of F, V& V will be transformed.

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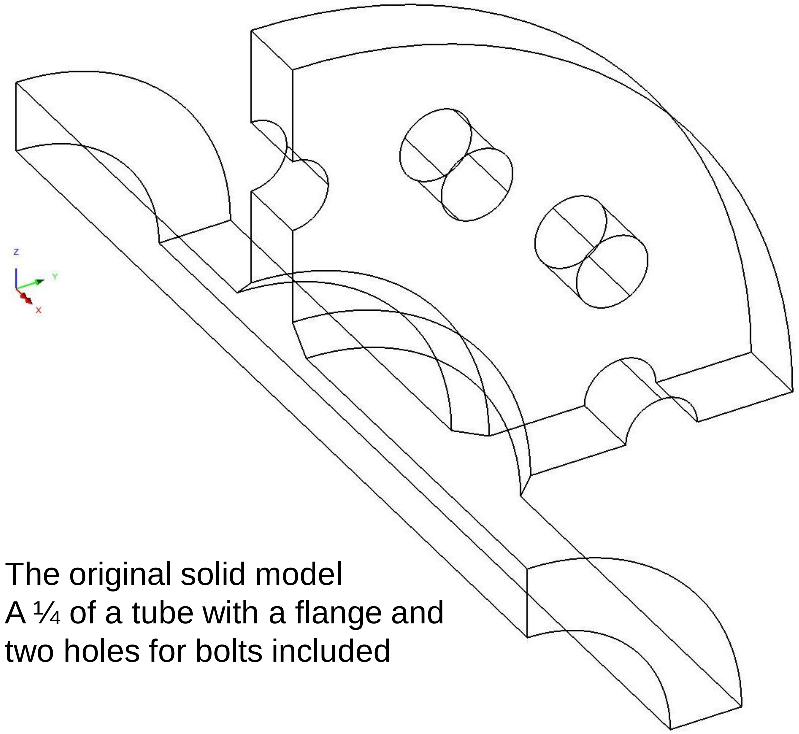
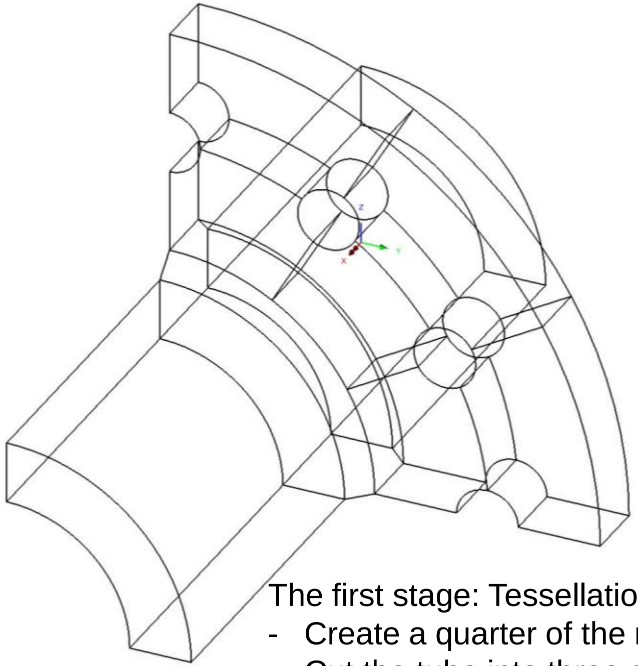
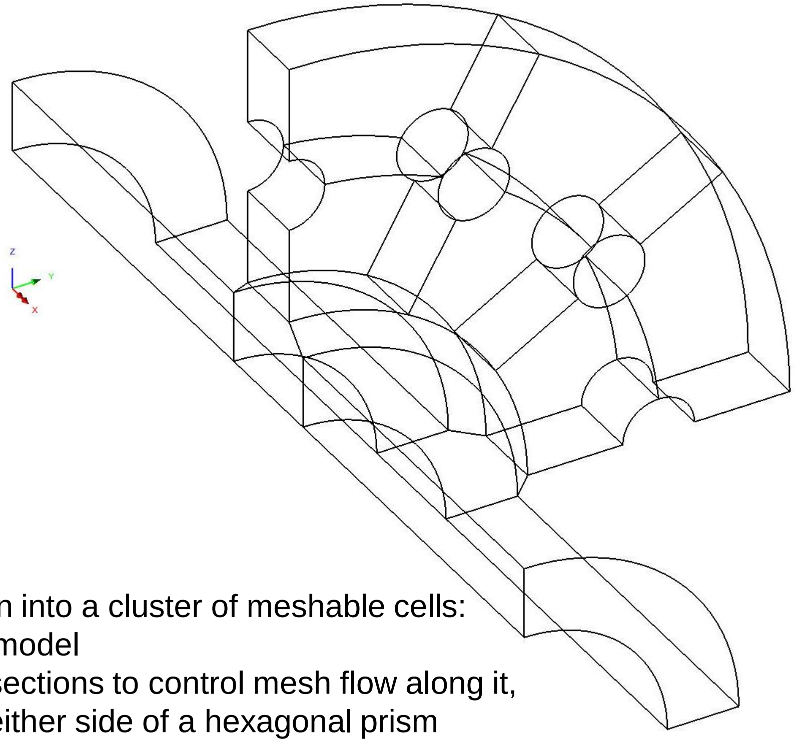
What about Formulation, Verification & Validation?

Validation and Verification in FEA: An Example



The original solid model
A ¼ of a tube with a flange and
two holes for bolts included





The first stage: Tessellation into a cluster of meshable cells:
- Create a quarter of the model
- Cut the tube into three sections to control mesh flow along it, a hexahedral prism on either side of a hexagonal prism
- Tessellate the flange into three pairs of hexagonal prisms

Tessellation can start from more complex shapes and follow instructions from an engineer who knows what a good hexahedral mesh looks like.
They may have particular requirements to the results, like collecting Results-of-Interest from particular points/curves/surfaces in the shape.

The first stage: Tessellation into a cluster of meshable cells:
The engineer want to express intent and let algorithms create the result, “Cut there, cut there, split along a circular arc, cut radially through the holes, etc.”
The software should keep a track on the shape representation underneath the graphics, and flag up when the resulting cells are meshable.
In this case all the cells are two kinds of prisms (a Hexagonal Prism is defined as meshable), and can be meshed when specifying Volume Mesh Controls: positions for the irregular edges which all are single five-way edges.

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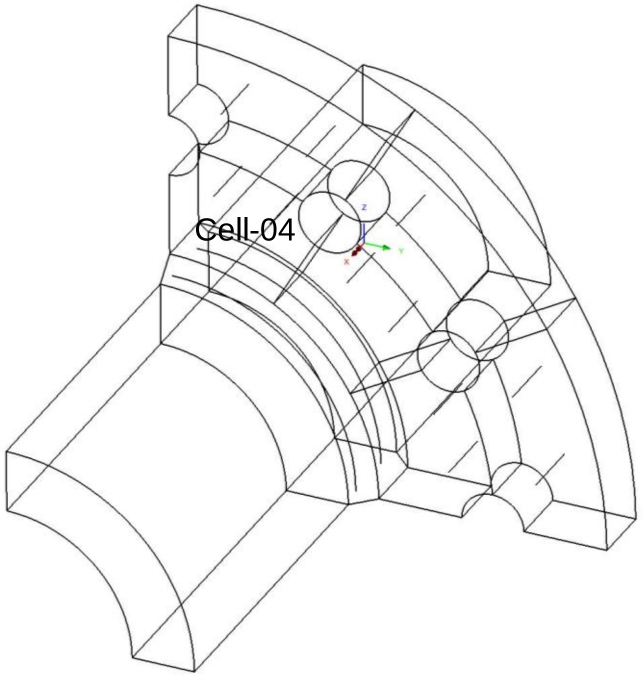
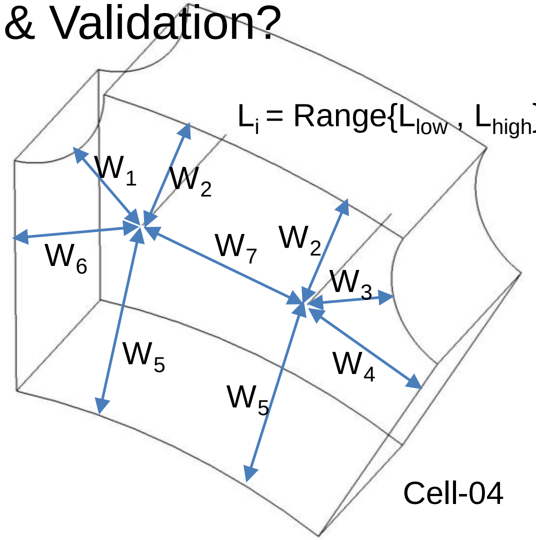
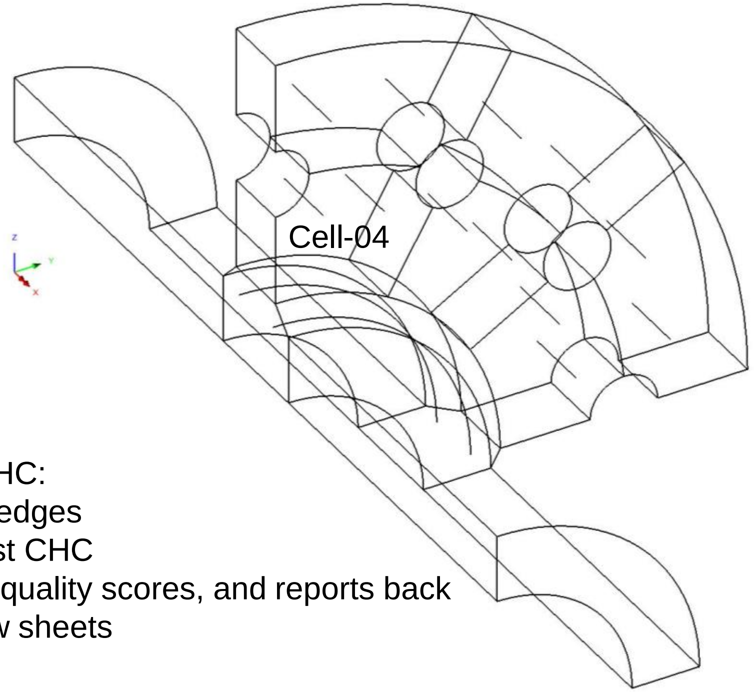
What about Formulation, Verification & Validation?







The second stage: Meshing into a cluster of cells, the CHC:

- The hexagonal prisms all need two five-way irregular edges
- The software suggest their positions and creates a first CHC
- The software evaluates the mesh and element shape quality scores, and reports back
- The user makes changes by altering the widths of flow sheets
- The user defines the rules for mesh refinements

The second stage: Meshing into a cluster of cells, the CHC:

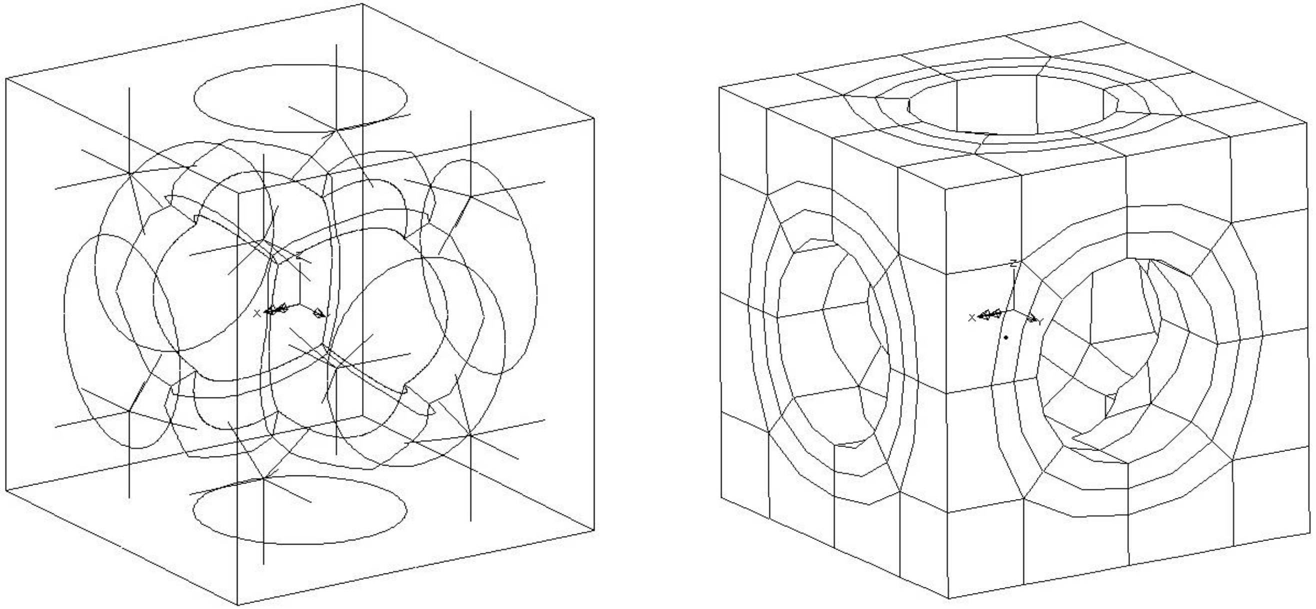
- The user augments the model (adding loads, boundary conditions, material, physical properties, analysis type, Results-of-Interest, target for accuracy, etc.),
- User defined criteria for element size ranges and element shape quality score thresholds will give ranges for the number of elements along each edge.
- From these ranges the system chooses suitable flow sheets widths to automatically create a first mesh, one of many possible CHCs.
- Where necessary, the user adjust the positions of the irregular edges by delta changes to the widths of the internal and external flow sheets and the mesh is ready.

The third stage: a convergence study, many small analyses:

- A sequence of analyses, a convergence study, can commence.
- The convergence study stops when the specified convergency criteria are met or the number of specified maximum number of analyses is reached.
- The results information for the Results-of-Interest can be accessed from the RDBMS database where these results from all the analyses are stored.

Dislocation Meshing - Discussion

Monty Roberts Willing Partners™: CASINOVA REDNECK



Irregular edge networks willing to deliver the best mesh they can do

The discovery of the significance of topology as the information carrier in meshing, such as the irregular vertices in polygons and irregular edge networks in polyhedra opens up a new way of specifying quadrilateral and hexahedral meshes.

The natural behaviour of quadrilateral and hexahedral meshes is embedded in algorithms for Tessellation and Meshing.
They will do what come naturally to them in the creation of high quality hexahedral meshes.

Now a user can populate the Volume Mesh Controls to position the irregular edges and internal vertices in the volume directly.
Dislocation Meshing is an “Inside-out” methodology. The external quadrilateral carpet and edge division numbers are only algorithm outputs.

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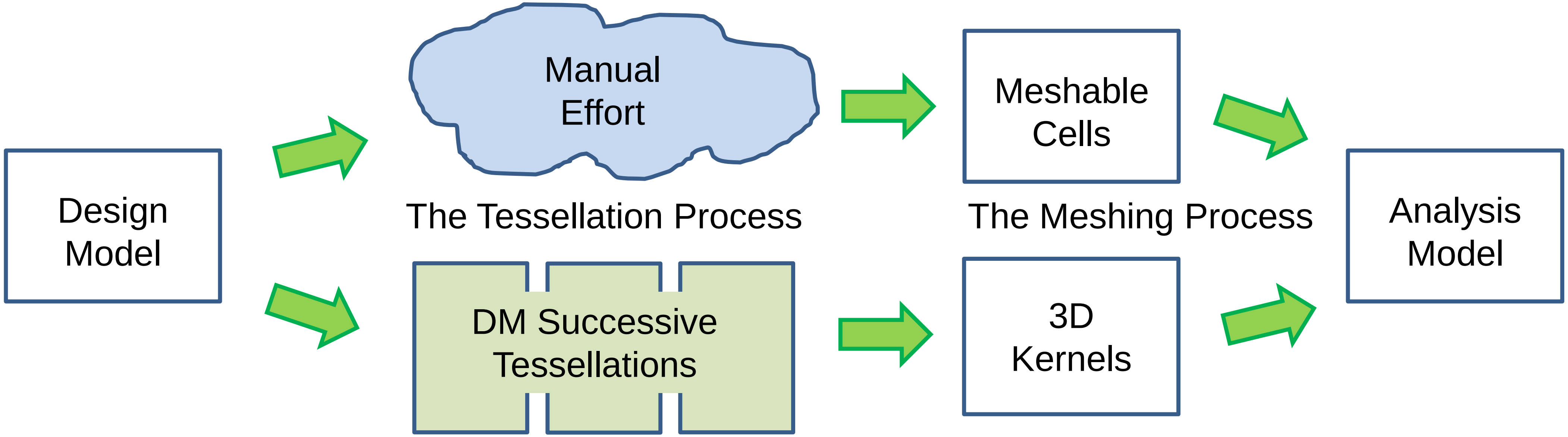
Dislocation Meshing - Discussion



The need for a tessellation process has prevented wide spread use of Hexahedral Meshing in the design process, because there was only one way of creating a hexahedral mesh: “hard graft”, the manual effort in breaking down the shape or rebuilding it into meshable cells demanded by the hexahedral meshing algorithms.
The rules for how to subdivide a shape into meshable cells were poorly understood, if known at all. As a consequence, the results from manual effort varies from one job to the next, and between analysts. Analysis sequences, one building on the next, become impractical. Tetrahedral meshing of solid models has been the fall-back position for decades.

The General Theory now offers an alternative:

- The understanding of how quadrilateral and hexahedral meshes behave
- The mathematical representation and the means to manipulate a cluster of cells using Topology Operators
  across the Tessellation and Meshing Processes. In fact, they become a continuous process where automation replace “hard graft”.

Dislocation Meshing - Discussion

Dislocation Meshing is founded on well-established mathematical theories and expands these to a natural mathematical language for representation and manipulation of surface and solid models using Cellular Non-Manifold Boundary-Representation Solid Model and Graph Theory notations.

Dislocation Meshing is a super-set meshing technology, spanning out a search space consisting of a substantial group of hexahedral meshes in a cluster of cells using both the Fundamental Set and the Auxiliary Sets of irregular edge networks. Within the complete space of alternatives, Dislocation Meshing is designed to focus on:

- the subset that uses irregular edge networks defined by the 3D Kernels only.
- the necessary number of irregular edges defined by the Fundamental Set.
- Including the Auxiliary Set as and when needed.

As a consequence, a large number of alternative irregular edge networks are excluded; mainly those with lower quality mesh flows and poorly shaped elements, often created by excessive use of the Auxiliary Set. In the remaining subset Dislocation Meshing can create a large number of hexahedral meshes with a minimum of irregular edge networks (the Fundamental Set), focusing on well-formed meshes with well-formed elements.

Other efforts in the domain of automatic hexahedral meshing can be measured by their effectiveness in creating any subset of these known solutions. Where these methods cannot create hexahedral meshes consistently (or not at all), the theory underpinning Dislocation Meshing can be used to explain why.

Dislocation Meshing - Conclusion

The patterns in the behaviour of quadrilateral and hexahedral meshes is a “Law of Nature”.

- Dislocation Meshing allows this natural behaviour of quadrilateral and hexahedral meshes to dominate the tessellation and meshing processes.
- The implementation of the patterns enables the meshes to willingly “Join-Up” with the user to create well-formed mesh flows and well-formed elements.

Dislocation Meshing is focused on offering flexibility in specifying the flow of a mesh. There are many alternatives represented in the Underdetermined Super Set Equation Systems and algorithms will direct the user towards the “most suitable” mesh:

- The algorithms make the choices, implement the tessellations and meshes and evaluate the quality of the mesh flow and element shapes. The algorithms report back to the engineer the scores of the chosen solution.
- The engineer guide and encourage the algorithms through a language they understand and eventually approves what the algorithms deliver.

There is a mathematical continuum from Parametric Design in CAD to result evaluation in FEA, and back, using a well-established mathematical notation that all structural engineers are familiar with, matrix manipulation using linear algebra.

The problem of “Dirty Geometry” should be eliminated by enabling designers and analysts to create correct shapes first time, again and again, no flukes.
The General Theory of Representation and Manipulation of Shapes has the potential to solve this problem once and for all.

Dislocation Meshing - Conclusion

Dislocation Meshing is an enabling technology and the missing link between the past and the future:

The Past:
- It can revive forgotten technologies like p- and hp-versions of FEA for accuracy ;
- It can revive sub-structuring for reduced clock wall-time when distributed across a cluster of HPC computers to solve large equation systems fast;

State of the Art:
- It fits within existing software architectures, FEA systems as well as CAD-systems;

The Future:
- It will fit into a Formulation, Verification & Validation framework and facilitate effective working methods
- It is an enabling technology to fully exploit the speed of emerging HPC technology, like Pica FLOPS machines, Exa FLOPS machines and whatever comes next.

What do you plan to do today so you are ready when Exa FLOPS machines arrive?

Dislocation Meshing - Conclusion

So, is Dislocation Meshing a Credible Solution to Automatic Hexahedral Meshing?

I leave it to you to make your mind up, not today, not tomorrow, but soon …

- Understand the patterns of behaviour that are presented here;
- Make your own sketches and drawings to explore what happens;
- Check out your own meshes;
- Implement the C-Core algorithm to work in your own pre-processor
- Ask questions for clarity;
- When you are convinced, contact us to discuss

How you can contribute to make these findings into robust commercial grade software

"Sometimes it's the people no one imagines anything of who do the things that no one can imagine."

Dr Alan Turing , Father of Modern Computing, who was in a 2019 public vote by the BBC's Icons-series named "The Greatest Person of the 20th Century".

Dislocation Meshing - Conclusion

"I guess you're not ready for *that*. But your kids are gonna LOVE it."

**Marty McFly, ending his guitar solo at
the Enchantment Under the Sea dance Nov 12 1955
in “Back to the Future”, cult movie from 1985**